A Novel Method for Fuzzy Measure Identification

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Abstract

Fuzzy measure and Choquet integral are effective tools for handling complex multiple criteria decision making (MCDM) problems in which criteria are inter-dependent. The identification of a fuzzy measure requires the determination of $2^n - 2$ values when the number of criteria is n. The complexity of this problem increases exponentially, which makes it practically very difficult to solve. Many methods have been proposed to reduce the number of values to be determined including the introduction of new special fuzzy measures like the λ-fuzzy measures. However, manipulations of the proposed methods are difficult from the aspects of high data complexity as well as low computation efficiency. Thus, this paper proposed a novel fuzzy measure identification method by reducing the data complexity to $n(n-1)/2$ and enhancing the computation efficiency by leveraging a relatively small number of variables and constraints for linear programming. The proposed method was developed based on the evaluation of pair-wise additivity degrees or interdependence coefficients between the criteria. Depending on the information being provided by decision-makers on the individual density of each criterion, the fuzzy measure can be constructed by solving a simple system of linear inequalities or a linear programming problem. This novel method is validated through a supplier selection problem which occurs frequently in real-world decision-making problems. Validation results demonstrate that the newly-proposed method can model real-world MCDM problems successfully.

Keywords: Choquet integral, Fuzzy integral, fuzzy measure, identification, linear programming, multiple criteria decision-making (MCDM).

1. Introduction

Fuzzy measures and Choquet integral are effective tools for handling the interactions and complexities of decision problems [1-8]. In multiple criteria decision making (MCDM) problems, fuzzy measures are used to represent the interactions between criteria, namely, the aspects of independences, complementarities and redundancies between criteria. Once a fuzzy measure is identified, a fuzzy integral can be used as an aggregation tool for evaluating and ranking alternatives [2, 4]. The Choquet Integral is one of the most used fuzzy integrals.

However, the identification of a fuzzy measure is one of the most difficult steps in applying fuzzy integrals to solve MCDM problems since $2^n - 2$ values of the fuzzy measure should be provided by the decision-maker(s) for an MCDM problem with n criteria. For example, if the number of criteria (n) is 7 (i.e., $n = 7$), the number of values of the fuzzy measure to be provided is 126. Therefore, for a large number of criteria, it becomes practically impossible for a decision-maker to provide the necessary data for fuzzy measure identification. Consequently, various fuzzy measure identification methods have been proposed. Typical examples can be found in the literature [1, 5-7, 9-16]. In [1, 4], the fuzzy measure identification methods are classified into three groups. The first group of methods is based on semantic considerations for guessing the fuzzy measure. The second group, called learning methods, is based on identification of the fuzzy measure by optimization methods. The third group of methods combines semantic and learning methods [17]. With the third group of methods, semantic considerations can be used to help reduce the large number ($2^n - 2$) of constraints in the quadratic optimization problems to be involved in the learning methods. Furthermore, it is worth noting that for some extremely complex decision problems, the required knowledge for fuzzy measure identifications may not be easily given by a decision-maker or he/she may not be knowledgeable enough about the decision problem. To resolve this kind of problems, Kojadinovic [18] proposed an unsupervised identification method based on the estimation of the fuzzy measure coefficients by means of information-theoretic functions. The approach mainly consists in replacing the rather subjective notion of importance of a subset of criteria by that, probabilistic, of information content of a subset of criteria, which can be estimated from the set of profiles. However, from a practical perspective, a sufficiently large number of profiles is obviously necessary (the number has to grow al-

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most exponentially with the number of criteria) to obtain accurate estimates of the fuzzy measure coefficients and therefore of the Choquet integral.

With the aim of reducing the complexity of initial data requirements for fuzzy measure identification, several subfamilies of fuzzy measures have been defined. In these families, some extra restrictions are added in order to decrease the number of coefficients while, keeping the modeling capabilities of the measures at the same time. Sugueno's [5] λ-fuzzy measure is such a specific fuzzy measure. Several methods [19-21] have been proposed for λ-fuzzy measure identifications. Since the proposed method in this paper is basically an optimization method, existing methods for fuzzy measure identification based on optimization approaches are reviewed further in details. In a recent review by Grabisch et al. [22], the main approaches, namely, least square, minimum split and minimum variance approaches, to fuzzy measure identification based on the Möbius transform of a capacity and k-additivity are reviewed and their advantages and inconveniences are discussed. The least square based approach is historically the first approach that has been proposed, it can be regarded as a generalization of classical multiple linear regression. The aim is to minimize the average quadratic distance between the overall utilities computed by means of the Choquet integral and the desired overall scores provided by the decision-maker. The weakness of this type of methods is that the obtained solution may not be unique because the objective function of the solved quadratic optimization problem may not be strictly convex. The maximum split approach is based on linear programming. Roughly speaking, the idea of the proposed approach is to maximize the minimal difference between the overall utilities of objects that have been ranked by the decision-maker(s) through the partial weak order. The advantage of this approach is its simplicity. However, it does not necessarily lead to a unique solution. The idea of the minimum variance method is to favor the "least specific" capacity, if any, compatible with the initial preferences of the decision-maker. One of the advantages of this approach is that it leads to a unique solution. Also, in the case of "poor" initial preferences involving a small number of constraints, this unique solution will not exhibit too specific behaviors characterized, for instance, by very high positive or negative interaction indices or a very uneven Shapley value.

Another optimization approach based on genetic algorithms has been proposed to reduce the complexity of fuzzy measure identification. For the purpose of ease of use, Lee and Leekwang [23] developed an identification method of λ-fuzzy measures based on genetic algorithms. From their results, even if there was no complete information for a fuzzy measure value of an element from the data set, their proposed method could still be applied for identification. It performed better than those presented in the works by Sekita [20], Tahani and Keller [19] as well as Wierzchon [21]. However, according to their identification procedure, most of the human-provided measure values are required for better solution quality. Namely, their method does not have a scheme to control the amount of lost information on the premise of producing a satisfactory solution. To overcome the difficulty of data collection for subjective importance identification, Sekita and Tabata [24] dropped the subsets whose grades of importance are near zero. This means reducing the cardinality of a set from n to $n-1$. Nevertheless, the complexity of data collection is $\mathrm{O}(2^n)$ and this improvement is slightly effective. Wang and Chen [14, 25] used a sampling process of subsets of a finite set and a procedure of data collection through experimental design methods. Based on this sampling procedure, they use genetic algorithms for effectively identifying λ-fuzzy measures. They reduced the complexity of data collection to $\mathrm{O}(2^n/n)$.

In order to reduce the complexity of initial data being required for fuzzy measure identification, Takahagi [12] proposed an approach based on two types of pair-wise comparison. The first one is based on the pair-wise comparison values of interaction degrees between criteria. The second one is based on the pair wise comparison values of weights of criteria. Then the fuzzy measure can be identified by the diamond pair-wise comparison and the φ_s transformation. Thus, the complexity of data collection can be reduced to $n(n-1)$.

To further reduce the initial data collection complexity and simplify the process of computation in fuzzy measure identification, this paper intends to propose a novel fuzzy measure identification method. The major contributions of this proposed novel method are: the reduction of initial data requirements and the simplification of computations. 1) The initial data being required will be reduced to $n(n-1)/2$ (n stands for the number of criteria), the number of pair-wise assessments of interdependence or interaction between criteria to be provided by the decision-makers. Then, a fuzzy measure can be identified via the resolution of a system of linear inequalities. In case decision-makers provide fuzzy assessments of the individual density of each criterion for a MCDM problem, the number of data being required will be $n(n-1)/2+n$. In this case, the Zimmermann's method for solving fuzzy linear multi-objective problems [17] can be used to identify the fuzzy measure. Thus, from the aspect of initial data being required, the proposed method is better than the Takahagi's method which requires $n(n-1)$ data. 2) From the viewpoint of computation simplification, in comparison with the commonly used quadratic optimization problems in existing fuzzy measure identification methods (e.g. [21]), which generally involve a large number, ($2^n - 2$), of constraints, the newly proposed method is simpler. Indeed, it reduces to the resolution of a system of linear inequalities or linear programming problem based upon a considerably smaller number of constraints and variables.

The remainder of this paper is organized as follows. In Section 2, the definitions of a fuzzy measure and a λ-fuzzy measure will be reviewed. In Section 3, a novel method for fuzzy measure identification will be proposed. In Section 4, one application example of the novel method is demonstrated, using a supplier selection problem which happens frequently in the real-world. Section 5 is devoted to the discussion of the obtained results. Section 6 presents conclusions and recommendations for further study.

2. Fuzzy Measure

Sugeno [5] presented the theory of fuzzy measures and fuzzy integrals as a means of expressing fuzzy systems in 1974. He further proposed his theory for modeling human subjective evaluation processes. In this section, we will review the Sugeno's definitions [5] as a background for the novel fuzzy measure identification method that will be developed in Section 3.

Definition 2.1: Let X be a set. A set function $g(\cdot)$ defined on the set of the subsets of X, $\beta(X)$, is called a fuzzy measure if it satisfies the following properties:

Property 1: Boundary conditions

$g\colon \beta(X) \to [0,1]$, and $g(\varnothing) = 0$ and $g(X) = 1$;

Property 2: Monotonocity

$\forall A, B \in \beta(X)$, if $A \subseteq B$, then $g(A) \le g(B)$

Property 3: Continuity

If $F_k \in \beta(X)$ for $1 \le k < \infty$, and the sequence $\{F_k\}$ is monotonic (in the sense of inclusion), then $\lim_{k\to+\infty} g(F_k) = g(\lim_{k\to+\infty} F_k)$

Remark 2.1: It must be noted that, if X is finite, then Property 3 can be dropped. The following are three special cases of fuzzy measures [5]. A fuzzy measure $g(\cdot)$ is said to be

(a) additive if

for all $A, B \in \beta(X)$ such that $A \cap B = \varnothing$, $g(A \cup B) = g(A) + g(B)$,

(b) super-additive if

for all $A, B \in \beta(X)$ such that $A \cap B = \varnothing$, $g(A \cup B) \ge g(A) + g(B)$,

(c) sub-additive if

for all $A, B \in \beta(X)$ such that $A \cap B = \varnothing$, $g(A \cup B) \le g(A) + g(B)$, and

(d) λ-fuzzy measure if

$$g(A \cup B) = g(A) + g(B) + \lambda g(A) g(B),$$

$$\text{for all } A, B \in \beta(X) \text{ and } A \cap B = \varnothing \tag{1}$$

where $\lambda \in]-1, +\infty[$.

In (1), the fuzzy measure is based on the parameter λ, which describes the degree of additivity the fuzzy measure holds. We have three important types of λ-fuzzy measures [5]:

(d1) super-additive if

$\lambda > 0$, then $g_\lambda(A \cup B) > g_\lambda(A) + g_\lambda(B)$,

(d2) additive if

$\lambda = 0$, then $g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B)$ and

(d3) sub-additive if

$\lambda < 0$, then $g_\lambda(A \cup B) < g_\lambda(A) + g_\lambda(B)$.

If $X = \{x_1, x_2, ..., x_n\}$, i.e. X is finite, the fuzzy measure can be identified by the following formula [5]:

$$g_\lambda(\{x_1, x_2, \cdots, x_n\}) = \sum_{i=1}^{n} g_i + \lambda \sum_{i1=1}^{n-1} \sum_{i2=i1+1}^{n} g_{i1} g_{i2} + \cdots +$$

$$\lambda^{n-1} g_1 g_2 \cdots g_n = \frac{1}{\lambda} \left| \prod_{i=1}^{n} (1 + \lambda g_i) - 1 \right|, \text{ for } -1 < \lambda < \infty$$

where $g_i = g_\lambda(\{x_i\}),\ i = 1, \cdots, n$ defines the fuzzy density of the fuzzy measure g_λ. If the fuzzy densities $g_i = g_\lambda(\{x_i\}),\ i = 1, \cdots, n$ are given, then in the case $\sum_{i=1}^{n} g_i = 1$ we have $\lambda = 0$; while if $\sum_{i=1}^{n} g_i \ne 1$, the parameter λ can be calculated by solving the following equation:

$$1 + \lambda = \prod_{i=1}^{n} (1 + \lambda g_i). \tag{2}$$

Note that the difficulty for solving equation (2) increases with the number of criteria, *n*.

3. Fuzzy Measure with Variable Additivity Degree

In this section, we will propose a fuzzy measure with variable degree of additivity. That is, instead of having a unique additivity coefficient λ as in the case of the λ-fuzzy measure, the additivity degree depends upon the considered subsets as follows

for all $A, B \in \beta(X)$, such that $A \cap B = \varnothing$, $g(A \cup B) = g(A) + g(B) + \lambda_{AB}$.

Here, λ_{AB} is the degree of additivity between the subsets of criteria *A* and *B*. λ_{AB} belongs to the interval [0, 1]. Assume $\beta(X)$ is the set of all subsets of an attribute set $X = \{x_1, x_2, .., x_n\}$. The number λ_{AB} expresses the interdependence between the subsets of criteria *A* and *B*. Therefore, in the sequel, we will call λ_{AB} an interdependence degree or coefficient. Detailed steps for the fuzzy measure identification based on the proposed idea of variable additivity are introduced below.

Procedure 3.1: Fuzzy Measure Construction.

Step 1: For each arbitrary pair of different attributes, i and j, an interdependence coefficient, λ_{ij}, such that $0 \le \lambda_{ij} \le 1$, has to be determined. For this purpose, the authors suggest the following approaches:

(i) Based on his/her experiences and knowledge, the decision-maker may subjectively determine these coefficients as follows. First, the decision-maker defines an ordinal scale of interdependence, e.g., complete dependence, very strong dependence, strong dependence, medium dependence, weak dependence and no dependence between any two criteria by natural language. The decision-maker then associates a numerical scale to this ordinal scale, e.g., 1.0 for complete dependence, 0.90 for very strong dependence, 0.75 for strong positive dependence, 0.50 for medium dependence, 0.30 for weak dependence and 0.00 for no dependence. The decision-maker has to keep in mind that in the process of determining the interdependence coefficients, if the interdependence between two attributes or criteria is close to 1, one of the two criteria is redundant. The decision-maker has to drop the redundant criterion. It is up to the decision-maker to decide on the threshold $0 < \lambda^0 < 1$ at which the decision-maker will drop one of the two criteria or attributes x_i and x_j if $\lambda_{ij} > \lambda^0$, for any *i* and *j*. This means that at the end of the process of the interdependency coefficients determination, practically, we will have $0 \le \lambda_{ij} \le \lambda^0 < 1$ for any pair of x_i criteria and x_j among the remaining ones.

(ii) The decision maker determines these coefficients based on the opinions of experts in the areas related to the problem at hand.

(iii) The coefficient of determination, R^2, which is the square of the correlation coefficient [26] between two variables, will be used as an estimate of the coefficients of interdependence. Here, the correlation coefficient measuring the strength of correlation between two variables can be interpreted as a measure of interdependence. For example, assume that the attributes x_i and x_j stand for the price and product performance of a product, respectively. By taking one of the two variables, x_j, as a dependent variable and another variable, x_i, as the explanatory variable, then the coefficient of determination can be computed via the correlation coefficient and used as an estimate of λ_{ij} by using data on different prices and the corresponding product performances.

(iv) The decision-maker can use the diamond pair-wise comparison as explained in [12].

Step 2: We define the density of a subset of two criteria or attributes $\{x_i, x_j\}$, and the whole set of criteria *X*, by $g(\{x_i, x_j\}) = g_i + g_j + \lambda_{ij}$ and $g(X) = \sum_{x_i \in X} g_i + \max_{x_i, x_j \in X, i \ne j} \lambda_{ij}$, respectively. By definition, we should have

$$0 \le g(\{x_i, x_j\}) = g_i + g_j + \lambda_{ij} \le 1$$

and

$$g(X) = \sum_{x_i \in X} g_i + \max_{x_i, x_j \in X, i \ne j} \lambda_{ij} = 1.$$

Based on the coefficients of pair-wise interdependence, λ_{ij} being determined in Step 1 and the last two conditions, we can determine the individual densities $g(\{x_i\}) = g_i$, $i = 1,\ldots,n$, by solving the following system of inequalities:

$$0 \le g_i + g_j + \lambda_{ij} \le 1, \text{ for all } x_i \text{ and } x_j \text{ in } X,\ i \ne j,$$
$$g(X) = \sum_{x_i \in X} g_i + \max_{x_i, x_j \in X, i \ne j} \lambda_{ij} = 1 \qquad (3)$$
$$0 \le g_i \le 1,\ i = 1,\ldots, n.$$

It is easy to see that the system of inequalities (3) has a solution. Indeed, let x_{i^0}, x_{j^0} be two criteria such that $\lambda_{i^0 j^0} = \max_{x_i, x_j \in X, i \ne j} \lambda_{ij}$. Taking $g_{i^0} = 1 - \max_{x_i, x_j \in X, i \ne j} \lambda_{ij}$ and $g_i = 0$, for $i \ne i^0$, we get a solution to the system.

When it is possible, the decision-maker may participate in the determination of the individual densities $g(\{x_i\}) = g_i$, $i = 1, \cdots, n$, by using his/her experience and knowledge about the criteria at hand or by consulting experts. The decision-maker may give a fuzzy estimate on the value of the density g_i for each attribute x_i. Without any loss of generality and for ease of presentation, we assume that the decision-maker's fuzzy estimation for each density g_i to be a triangular fuzzy number $\tilde{g}_i = (a_i, m_i, b_i)$, $0 \le a_i \le m_i \le b_i \le 1 - \lambda_{ij}$. Then the determination of the densities g_i, $i = 1, \cdots, n$ of the fuzzy measure can be treated as a multi-objective problem by assuming that we are willing to maximize all the membership functions, $\mu_i(g_i)$, $i = 1, \cdots, n$, of the fuzzy densities $\tilde{g}_i$, $i = 1, \cdots, n$. Finally, by using Zimmermann's approach for solving the fuzzy linear programming problem [17], we get the following problem for the determination of the densities:

$$\text{Max } \alpha \qquad (4)$$

s.t. $\mu_i(g_i) \ge \alpha, \quad i = 1,...,n$

$$\sum_{i=1}^{n} g_i + \max_{\substack{x_i, x_j \in X \\ i \ne j}} \lambda_{ij} = 1$$

$$0 \le g_i + g_j + \lambda_{ij} \le 1,\ i, j \in \{1,2,...,n\},\ i \ne j,$$

$$\alpha \in [0,1],\ g_i \in [a_i, b_i],\ i = 1,...,n.$$

Here, α represents the α-cut level of the fuzzy numbers $\tilde{g}_i$, $i = 1,\cdots,n$. In fact, any set of densities that satisfies the constraints of the problem (4) can be taken as a solution. We look for the maximum value of α in this problem, because the larger the value α, the better the precision of the density estimates g_i, $i = 1,\cdots,n$ being found by solving the problem (4). It is easy to see that if we take α=0, $b_i = 1 - \max_{j \in \{1,...,n\} \setminus \{i\}} \lambda_{ij}$, $a_i = 0$ for all $i \in \{1,...,n\}$ in the problem (4), its system of constraints will be reduced to a system which is similar to that of problem (3). A solution to such a system of constraints can be found by the same way a solution was found for the system (3). Therefore, there exists a solution to the system of constraints of the problem (4). Thus, the problem (4) itself has a solution when $b_i = 1 - \max_{j \in \{1,...,n\} \setminus \{i\}} \lambda_{ij}$, $a_i = 0$ for all $i \in \{1,...,n\}$.

Step 3: The fuzzy measure is determined based upon the densities obtained from the resolution of the problem (4) or (3) using the following formula

$g(\{x_i, x_j\}) = g_i + g_j + \lambda_{ij}$ for all pairs of criteria $\{x_i, x_j\}$, $i \ne j$,

$$g(X) = \sum_{x_i \in X} g_i + \max_{x_i, x_j \in X, i \ne j} \lambda_{ij} = 1 \qquad (5)$$

$$g(A) = \sum_{x_i \in A} g_i + \max_{x_i, x_j \in A, i \ne j} \lambda_{ij}$$

for all subsets A of X such that Card(A)≥ 2 and $g(\varnothing) = 0$.

In the following proposition we show that the set function (5) is a fuzzy measure.

Proposition 3.1: The set function being defined in (5) is a fuzzy measure.

Proof: By construction, we have $g(\varnothing) = 0$ and $g(X) = 1$, that is, part of Property 1 of Definition 2.1 of a fuzzy measure is satisfied. It remains to prove that $g(A) \le 1$ for all subsets A of X such that $\mathrm{Card}(A) \ge 2$. Let us first prove Property 2 of Definition 2.1, that is, given two subsets A and B of X such that $A \subset B$, we have $g(A) \le g(B)$. Since $A \subset B$, then

$$\sum_{x_i \in A} g_i \le \sum_{x_i \in B} g_i \quad \text{and} \quad \max_{x_i, x_j \in A, i \ne j} \lambda_{ij} \le \max_{x_i, x_j \in B, i \ne j} \lambda_{ij}$$

Adding these two inequalities, we get

$$g(A) = \sum_{x_i \in A} g_i + \max_{x_i, x_j \in A, i \ne j} \lambda_{ij} \le$$

$$g(B) = \sum_{x_i \in B} g_i + \max_{x_i, x_j \in B, i \ne j} \lambda_{ij}$$

Now let us prove that, for any subset A of X, we have $0 \le g(A) \le 1$ for complete satisfaction of Property 1 of Definition 2.1. If the subset A contains one or two elements, the inequality $0 \le g(A) \le 1$ is satisfied according to the constraints of problem (3) or (4). Assume now that A contains more than three criteria. By Definition 2.1, we have

$$g(A) = \sum_{x_i \in A} g_i + \max_{x_i, x_j \in A, i \ne j} \lambda_{ij}.$$

Therefore, $g(A) \ge 0$. Finally, since $A \subset X$, according to Property 2 of Definition 2.1, $g(A) \le g(X) = 1$. This completes the proof.

4. Application example of the Novel Fuzzy Measure Identification Method in Real-World MCDM Problems

The provider selection problem is one of the most commonly discussed management issues by both management scholars and real world managers. In this paper, the provider selection problem being introduced by Inouei and Amagasa [27] will be used to verify the feasibility of the novel method in real-world MCDM problems.

Service quality (x_1), price (x_2), customer satisfaction (x_3) and product performance (x_4) are the four most commonly used criteria for evaluating a provider. The above-mentioned four criteria are usually assumed to be independent of each other for the convenience of calculation. However, in the real world, we may find that service quality (x_1) influences product price (x_2) and customer satisfaction (x_3). Product price (x_2) may influence customer satisfaction (x_3) and product performance (x_4) (since higher cost and thus, higher price products usually have better performance). Better quality products (x_1) may be priced (x_2) higher. Product performance (x_4) and price (x_2) may influence each other, since a better performance product may be priced as a higher market segmentation product; while higher cost, and thus higher price products sometimes exhibit better design and, thus, better performance (x_4). Product performance (x_4) also may influence customer satisfaction (x_3) directly. The dependence structure of this sample MCDM problem is drawn below in Figure 1 as a reference for readers' better understanding. Based upon the rationale and relationships mentioned above, we may find that assuming in-dependence of the criteria is far from being realistic, and may be misleading when such a decision problem structure is used to guide real world managers' decisions.

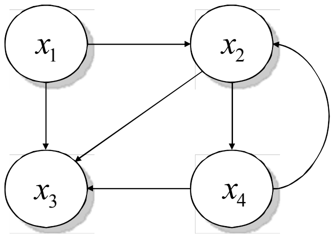

Figure 1. Structure of the Dependence of Criteria in the MCDM Example.

Applying Procedure 3.1 for an identification of the fuzzy measure, we first assume that the decision maker provides the following pair-wise interdependence degrees between criteria:

$$\lambda_{12}=0.03,\ \lambda_{13}=0.04,\ \lambda_{14}=0.03,$$
$$\lambda_{23}=0.06,\ \lambda_{24}=0.10 \ \text{ and } \ \lambda_{34}=0.03.$$

By symmetry, we get

$$\lambda_{21}=0.03,\ \lambda_{31}=0.04,\ \lambda_{41}=0.03,$$
$$\lambda_{32}=0.06,\ \lambda_{42}=0.10 \ \text{ and } \ \lambda_{43}=0.03\ .$$

Meanwhile, assume that the decision-maker provides the fuzzy estimates of the individual densities g_i, $i=1,2,...,4$ as the following triangular fuzzy numbers

$$\tilde{g}_1=(a_1,m_1,b_1)=(0.01,0.25,0.96),$$
$$\tilde{g}_2=(a_2,m_2,b_2)=(0.01,0.25,0.90),$$
$$\tilde{g}_3=(a_3,m_3,b_3)=(0.03,0.25,0.94),$$

and

$$\tilde{g}_4=(a_4,m_4,b_4)=(0.01,0.15,0.94)\,.$$

One may easily verify that $\alpha=1$ is an optimal solution of the linear programming problem (4) with g_1=0.25, g_2=0.25, g_3=0.25 and g_4=0.15. Indeed, for the value $\alpha=1$, the first constraint of problem (5) implies $\mu_i(g_i)=1$, $i=1,...,4$. Hence g_1= 0.25, g_2=0.25, g_3=0.25 and g_4=0.15.

We have $\max\limits_{i,j\in\{1,..4\},\ i\neq j}\lambda_{ij}=0.1$, then the constraints of problem (4) become

$$0\le g_1+g_2\le 0.97\ ,$$
$$0\le g_1+g_3\le 0.96\,,$$
$$0\le g_1+g_4\le 0.97\ ,$$
$$0\le g_2+g_3\le 0.94\,,$$
$$0\le g_2+g_4\le 0.90\ ,$$
$$0\le g_3+g_4\le 0.97\ ,$$
$$\sum_{i\in\{1,2,3,4\}} g_i+\max_{\substack{x_i,x_j\in X\\ i\neq j}}\lambda_{ij}=1$$

Thus all the constraints of the problem (4) are satisfied. Hence, according to the formula (5), the set function defined by $g(A)=\sum\limits_{x_i\in A} g_i+\max\limits_{x_i,x_j\in A,\,i\neq j}\lambda_{ij}$ for all subsets A of $\{x_1,x_2,x_3,x_4\}$ containing at least two criteria is a fuzzy measure. In the following computations, we denote the attributes x_1,x_2,x_3 and x_4 by 1, 2, 3 and 4 respectively.

$$g(\{1,2\})=g_1+g_2+\lambda_{12}=0.25+0.25+0.03=0.53\ ,$$
$$g(\{1,3\})=g+g+\lambda_{13}=0.25+0.25+0.04=0.54a\ ,$$
$$g(\{1,4\})=g_1+g_4+\lambda_{14}=0.25+0.15+0.03=0.43\ ,$$
$$g(\{2,3\})=g_2+g_3+\lambda_{23}=0.25+0.25+0.06=0.56\ ,,$$
$$g(\{2,4\})=g_2+g_4+\lambda_{24}=0.25+0.15+0.10=0.50\ ,$$
$$g(\{3,4\})=g_3+g_4+\lambda_{34}=0.25+0.15+0.03=0.43\ ,$$
$$g(\{1,2,3\})=g_1+g_2+g_3+\max\{\lambda_{ij},i,j=1,2,3,\ \ i\neq j\}$$
$$=0.25+0.25+0.25+0.06=0.81,$$
$$g(\{1,2,4\})=g_1+g_2+g_4+\max\{\lambda_{ij},i,j=1,2,4,\ \ i\neq j\}$$
$$=0.25+0.25+0.15+0.10=0.75,$$
$$g(\{1,3,4\})=g_1+g_3+g_4+\max\{\lambda_{ij},i,j=1,3,4,\ \ i\neq j\}$$
$$=0.25+0.25+0.15+0.04=0.69,$$
$$g(\{2,3,4\})=g_2+g_3+g_4+\max\{\lambda_{ij},i,j=2,3,4,\ \ i\neq j\}$$
$$=0.25+0.25+0.15+0.10=0.75,$$

and

$$g(\{1,2,3,4\})=g_1+g_2+g_3+g_4$$
$$+\max\{\lambda_{ij},i,j=1,2,3,4,\ \ i\neq j\}$$
$$=0.25+0.25+0.25+0.15+0.10=1.00.$$

Assume that four providers, *A*, *B*, *C* and *D*, are available as alternatives for evaluations. Scores of the providers, based upon the four given criteria x_1,x_2,x_3 and x_4, are given below in Table 1.

Table 1. Scores of the providers based on the four given criteria.

Alternatives	SERVICE QUALITY	Price	Customer	Product Performance
A	55	85	85	45
B	55	45	75	85
C	85	75	55	65
D	75	65	65	75

Let us now recall the definition of the Choquet integral. Let $g(\cdot)$ be a fuzzy measure defined on a finite set $X=\{x_1,x_2,...,x_n\}$ and $h:X\to[0,1]$ be a function representing the evaluation of $x_1,x_2,...,x_n$ such that $h(x_1)\ge h(x_2)\ge...\ge h(x_n)$. Then the Choquet integral of the function $h(\cdot)$ is given by the following equation [5, 7, 28-37]:

$$(C)\int hg = h(x_n)g(H_n) + [h(x_{n-1}) - h(x_n)]g(H_{n-1}) + \ldots + [h(x_1) - h(x_2)]g(H_1) \quad (6)$$

where $H_i = \{x_1, x_2, \ldots, x_i\}$.

The given alternatives can be evaluated by Choquet Integral (6). To normalize the values in TABLE I, we divide each number by 100, so as to assure that all the numbers lie within the interval [0,1], as required in the Choquet integral. Then, evaluation results can be reached.

For provider A, $h(x_2) \geq h(x_3) \geq h(x_1) \geq h(x_4)$. The Choquet integral of $h(\cdot)$ can be calculated by (6) as

$$\begin{aligned}(C)\int hg &= h(x_4)g(H_4) + [h(x_1) - h(x_4)]g(H_3) \\ &\quad + [h(x_3) - h(x_1)]g(H_2) + [h(x_2) - h(x_3)]g(H_1) \\ &= 0.45 \times 1 + [0.55 - 0.45] \times 0.81 + [0.85 - 0.55] \times 0.53 \\ &\quad + [0.85 - 0.85] \times 0.25 = 0.70\end{aligned}$$

For provider B, $h(x_4) \geq h(x_3) \geq h(x_1) \geq h(x_2)$. The Choquet integral of $h(\cdot)$ can be calculated by (6) as

$$\begin{aligned}(C)\int hg &= h(x_2)g(H_4) + [h(x_1) - h(x_2)]g(H_3) \\ &\quad + [h(x_3) - h(x_1)]g(H_2) + [h(x_4) - h(x_3)]g(H_1) \\ &= 0.45 \times 1 + [0.55 - 0.45] \times 0.81 + [0.75 - 0.55] \times 0.53 \\ &\quad + [0.85 - 0.75] \times 0.25 = 0.662\end{aligned}$$

For provider C, $h(x_1) \geq h(x_2) \geq h(x_4) \geq h(x_3)$. The Choquet integral of $h(\cdot)$ can be calculated by (6) as

$$\begin{aligned}(C)\int hg &= h(x_3)g(H_4) + [h(x_4) - h(x_3)]g(H_3) \\ &\quad + [h(x_2) - h(x_4)]g(H_2) + [h(x_1) - h(x_2)]g(H_1) \\ &= 0.55 \times 1 + [0.65 - 0.55] \times 0.81 + [0.75 - 0.55] \times 0.53 \\ &\quad + [0.85 - 0.75] \times 0.25 = 0.709\end{aligned}$$

Finally, for provider D, $h(x_4) \geq h(x_1) \geq h(x_3) \geq h(x_2)$. The Choquet integral of $h(\cdot)$ can be calculated by (6) as

$$\begin{aligned}(C)\int hg &= h(x_2)g(H_4) + [h(x_3) - h(x_2)]g(H_3) \\ &\quad + [h(x_1) - h(x_3)]g(H_2) + [h(x_4) - h(x_1)]g(H_1) \\ &= 0.65 \times 1 + [0.65 - 0.65] \times 0.81 + [0.75 - 0.65] \times 0.53 \\ &\quad + [0.75 - 0.75] \times 0.25 = 0.703\end{aligned}$$

Based upon the obtained values, we can rank the providers as, $C \succ D \succ A \succ B$ which implies that provider C is the most favorable provider.

Remark 4.1: It is recommended to start solving the problem (4) by taking $\alpha = 1$.

5. Discussions

$\lambda-$fuzzy measure is one of the most popular fuzzy measures used in MCDM. In this paper we have presented an alternative to this fuzzy measure. It is based on the evaluation of the pair-wise interdependence between the criteria. In the introduced novel fuzzy measure identification method, the decision-maker is also given the opportunity to provide a fuzzy assessment of the fuzzy densities. The actual fuzzy density of each criterion is found via the resolution of the system of linear inequalities (3) or the linear programming problem (4). Then, the density of the other subsets of criteria can be calculated using the formula (5).

Meanwhile, compared to other fuzzy measure identification methods, including λ-fuzzy measure identification, the proposed method is quite simple, since it requires the decision-maker to make a pair-wise assessment of the interdependence coefficient between criteria and a fuzzy assessment of the densities. In Procedure 3.1 we have provided some ways of assessment of the pair-wise interdependence coefficients.

To demonstrate the above-mentioned advantages, we examine the same example treated in Section 4, using the traditional $\lambda-$fuzzy measure identification method. Assume that the AHP pair-wise comparison matrix between the criteria is

$$H = \begin{bmatrix} 1 & 1/3 & 1/3 & 1/2 \\ 3 & 1 & 1/2 & 1/4 \\ 3 & 2 & 1 & 1 \\ 2 & 4 & 1 & 1 \end{bmatrix}$$

First, initial weights, $(0.110,\ 0.179,\ 0.326,\ 0.384)$, can be obtained by using AHP. Let the fuzzy measure weights be $\boldsymbol{w} = c(0.110,\ 0.179,\ 0.326,\ 0.384)$, where c is a positive number; i.e.,

$$w_1 : g_\lambda(\{x_1\}) = 0.110c,\quad w_2 : g_\lambda(\{x_2\}) = 0.179c,$$
$$w_3 : g_\lambda(\{x_3\}) = 0.326c,\quad w_4 : g_\lambda(\{x_4\}) = 0.384c,$$

and

$$g_\lambda(X) = g_\lambda(\{x_1, x_2, x_3, x_4\} = 1,$$
$$g_\lambda(X) = \sum_i w_i + \lambda \sum_{i,j>i} w_i w_j + \lambda^2 \sum_{i,j,k} w_i w_j w_k + \lambda^3 w_1 w_2 w_3 w_4.$$

By assuming $\lambda = 3$, we get $c = 0.569$ (refer Inouei and Amagasa [27]). A fuzzy measure characterized by the following values can be obtained: $g_\lambda(\{x_1\})$ =0.063, $g_\lambda(\{x_2\}) = 0.102$, $g_\lambda(\{x_3\}) = 0.186$, $g_\lambda(x_4) = 0.219$, $g_\lambda(\{x_1, x_2\}) = 0.184$, $g_\lambda(\{x_1, x_3\}) = 0.283$, $g_\lambda(\{x_1, x_4\}) = 0.322$, $g_\lambda(\{x_2, x_3\}) = 0.344$, $g_\lambda(\{x_2, x_4\}) = 0.388$, $g_\lambda(\{x_3, x_4\}) = 0.526$, $g_\lambda(\{x_1, x_2, x_3\}) = 0.427$, $g_\lambda(\{x_1, x_2, x_4\}) = 0.523$, $g_\lambda(\{x_1, x_3, x_4\}) = 0.688$, $g_\lambda(\{x_2, x_3, x_4\}) = 0.789$. Then $g_\lambda(H_1) = g_\lambda(\{x_1\}) = 0.063$, $g_\lambda(H_2) = g_\lambda(\{x_1, x_2\}) = 0.184$, $g_\lambda(\{H_3\}) = g_\lambda(\{x_1, x_2, x_3\}) = 0.427$ and $g_\lambda(H_4) = g_\lambda(X) = 1$.

Based upon the fuzzy measure obtained, we can compute the Choquet integral for providers *A*, *B*, *C* and *D* by (5), as follows.

For Provider *A*, we have $h(x_2) \ge h(x_3) \ge h(x_1) \ge h(x_4)$, then the Choquet integral of *h(.)* is

$$(C)\int hg = h(x_4)g(H_4) + [h(x_1) - h(x_4)]g(H_3) + [h(x_3) - h(x_1)]g(H_2) + [h(x_2) - h(x_3)]g(H_1),$$

$$(C)\int hg = 0.45 \times 1 + [0.55\text{-}0.45] \times 0.427 + [0.85\text{-}0.55] \times 0.184 + [0.85\text{-}0.85] \times 0.063 = 0.548.$$

For Provider *B*, we have $h(x_4) \ge h(x_3) \ge h(x_1) \ge h(x_2)$, then the Choquet integral of $h(\cdot)$ is

$$(C)\int hg = h(x_2)g(H_4) + [h(x_1) - h(x_2)]g(H_3) + [h(x_3) - h(x_1)]g(H_2) + [h(x_4) - h(x_3)]g(H_1),$$

$$(C)\int hg = 0.45 \times 1 + [0.55 - 0.45] \times 0.427 + [0.75 - 0.55] \times 0.184 + [0.85 - 0.75] \times 0.063 = 0.536.$$

For Provider *C*, we have $h(x_1) \ge h(x_2) \ge h(x_4) \ge h(x_3)$; then the Choquet integral of $h(\cdot)$ is

$$(C)\int hg = h(x_3)g(H_4) + [h(x_4) - h(x_3)]g(H_3) + [h(x_2) - h(x_4)]g(H_2) + [h(x_1) - h(x_2)]g(H_1),$$

$$(C)\int hg = 0.55 \times 1 + [0.65 - 0.55] \times 0.427 + [0.75 - 0.65] \times 0.184 + [0.85 - 0.75] \times 0.063 = 0.617.$$

For Provider *D*, we have $h(x_4) \ge h(x_1) \ge h(x_3) \ge h(x_2)$, then the Choquet integral of *h(.)* is

$$(C)\int hg = h(x_2)g(H_4) + [h(x_3) - h(x_2)]g(H_3) + [h(x_1) - h(x_3)]g(H_2) + [h(x_4) - h(x_1)]g(H_1),$$

$$(C)\int hg = 0.65 \times 1 + [0.65 - 0.65] \times 0.427 + [0.75 - 0.65] \times 0.184 + [0.75 - 0.75] \times 0.063 = 0.668.$$

Table 2. Scores and ranks obtained based on the proposed novel and traditional fuzzy measure identification methods.

Alternatives	NOVEL METHOD		Traditional method[17]	
	RANK	Fuzzy Integral	Rank	Fuzzy Integral
A	3	0.700	3	0.548
B	4	0.622	4	0.536
C	1	0.709	2	0.617
D	2	0.703	1	0.668

The scores and ranks obtained based upon the proposed novel and traditional λ-fuzzy measure identification methods are summarized below in Table 2. By the proposed method, provider *C* is the most favorable provider. On the other hand, using traditional fuzzy measure identification, the preference of providers is ordered as $D \succ C \succ A \succ B$, which implies that provider *D* is the most favorable provider. From this comparison, we can conclude that the proposed method can enhance the traditional method and lead to new results.

In most existing fuzzy measure identifications, $2^n - 2$ subjective estimates ($g(A), A \subset X, A \ne X, A \ne \varnothing$) must be determined and the resolution of a quadratic optimization problem or (2) is required. As mentioned in the introduction, as far as we know, the best method [12] requires $n \times (n-1)$ data. Meanwhile, with our method, a maximum of $n \times (n-1)/2$ subjective estimates are needed for the determination of the pair-wise interdependence between criteria λ_{ij}, $i \ne j$ (or $n \times (n-1)/2 + n$ estimates when the decision-maker is able to provide the fuzzy estimation of the fuzzy densities $g(x_i)$, $i = 1, \cdots, n$). In addition, from computational point of view, generally, existing optimization fuzzy measure identification methods involve non linear objective functions and a large number (generally $2^n - 2$) of constraints. Our method is computationally simpler in the sense that by solving the system (3) (or the linear programming problem (4)) and using formula (5), the fuzzy measure is completely identified.

Now we show that the fuzzy measure identified using the proposed fuzzy measure identification method is not a $\lambda -$fuzzy measure in general. As an example, we will show that the fuzzy measure identified in the MCDM application of Section 4 cannot be identified as a $\lambda -$fuzzy measure. Thus, it is not a $\lambda -$fuzzy measure. Let us assume, by contrast, that there exists $\lambda^0 \in [-1, +\infty[$ such that the fuzzy measure identified in the MCDM application in Section 4 is a $\lambda^0 -$fuzzy measure. For computation convenience, we denote the attributes x_1, x_2, x_3 and x_4 by 1, 2, 3 and 4 respectively.

For criteria 1 and 2, we have $g_1 = 0.25$ and $g_2 = 0.25$. Thus, $g(\{1\} \cup \{2\}) = g_1 + g_2 + \lambda_{12} = 0.25 + 0.25 + 0.03 = 0.53$. By using (1), we will reach

$$g(\{1\} \cup \{2\}) = g_1 + g_2 + \lambda^0 g_1 g_2 = 0.25 + 0.25 + \lambda^0 \times 0.25 \times 0.25 = 0.53.$$

Thus, $\lambda^0 = (0.53 - 0.50)/0.0625 = 0.48$.

Similarly, for criteria 2 and 4,

$$g(\{2\} \cup \{4\}) = g_2 + g_4 + \lambda_{24} = 0.25 + 0.15 + 0.10 = 0.50.$$

By using (1), we will reach

$$g(\{2\} \cup \{4\}) = g_2 + g_4 + \lambda^0 g_2 g_4 = 0.25 + 0.15 + \lambda^0 \times 0.25 \times 0.15 = 0.50.$$

Thus, $\lambda^0 = (0.50 - 0.40)/(0.25 \times 0.15) = 2.67$.

From the calculation results, we see that λ^0 is not unique, as it must be with a $\lambda -$fuzzy measure. This contradiction means that the fuzzy measure obtained through our identification method is not a $\lambda-$fuzzy measure in general.

6. Conclusions

In this paper, a novel method for identifying fuzzy measures was proposed. The proposed method is practical and simple, requiring the decision-maker to provide $n(n-1)/2$ pair-wise comparisons of interdependence between criteria, and if possible a fuzzy evaluation of the densities $g(x_i),\ i=1,\cdots,n$ (individual importance or weight of each of the criteria). These two requirements are feasible and acceptable, from a practical point of view. From the viewpoint of computation, the proposed method reduces the fuzzy measure identification problem to a system of linear inequalities or a linear programming problem involving very few variables - the densities $g(x_i),\ i=1,\cdots,n$, and a limited number of constrains - which can be solved easily. Relative to other identification methods, which are usually solved by means of optimization approaches with a comparatively huge number of variables and constraints equal to 2^n-2 ($g(\varnothing)$, and $g(X)$ are known), the proposed method is simpler. We also have shown that the fuzzy measure identified via our method is not a λ-fuzzy measure, in general; so the fuzzy measure identified by our method is new. This fuzzy measure presents a potential for real-world applications. Exploring new ways of identifying such fuzzy measures is a worthy direction of research.

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