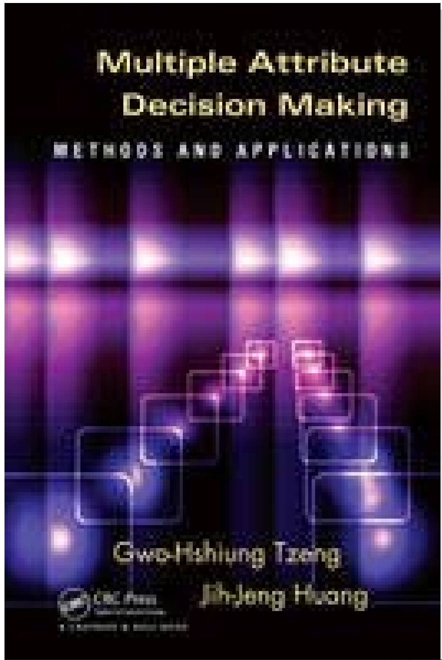
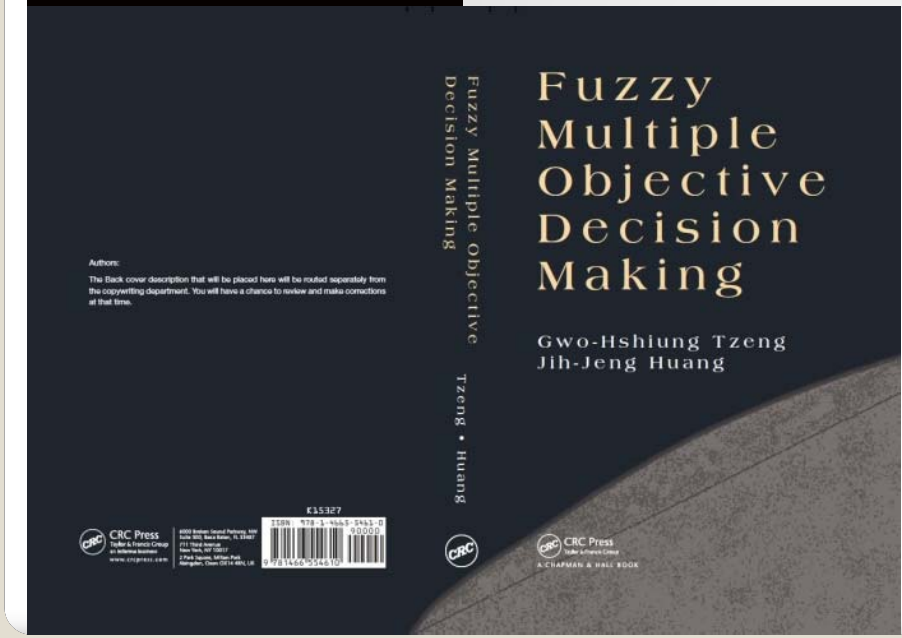


New Concepts and Trends of Hybrid MCDM For Tomorrow

(MRDM, MADM, and MODM)

- Solving Actual Problems for the Real World –

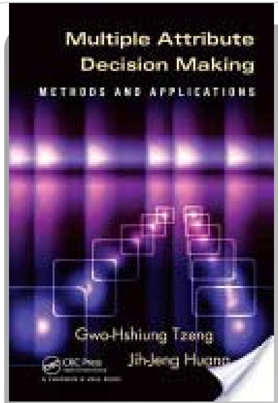
(Academic Speech)



Gwo-Hshiung Tzeng
Distinguished Chair Professor

Graduate Institute of Urban Planning
College of Public Affairs
National Taipei University
(NTPU)

Talk for KUAS-IEM
March 12, 2016 in Saturday, 2:00pm-4:30pm

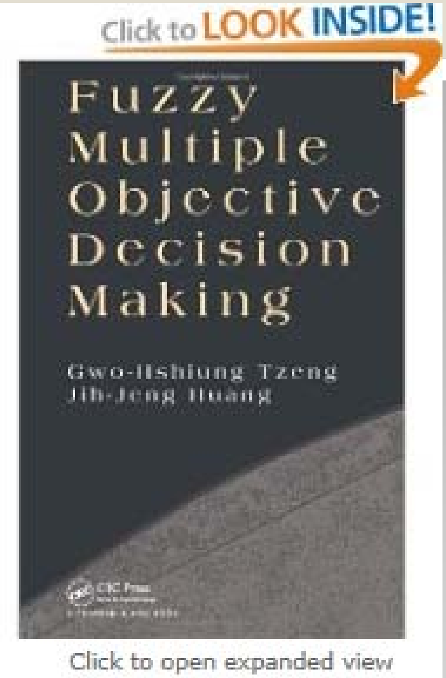


Two New Books (1)

Multiple Attribute Decision Making: Methods and Applications

Gwo-Hshiung Tzeng, Jih-Jeng Huang, CRC Press, Taylor & Francis Group, 2011, 349 pages

- Decision makers are often faced with several conflicting alternatives. How do they evaluate trade-offs when there are more than three criteria? To help people make optimal decisions, scholars in the discipline of multiple criteria decision making (MCDM) continue to develop new methods for structuring preferences and determining the correct relative weights for criteria. A compilation of modern decision-making techniques, Multiple Attribute Decision Making: Methods and Applications focuses on the fuzzy set approach to multiple attribute decision making (MADM). Drawing on their experience, the authors bring together current methods and real-life applications of MADM techniques for decision analysis. **They also propose a novel hybrid MADM model that adopts DEMATEL and DEMATEL-based ANP (DANP) with VIKOR procedures**.
- **The first part of the book focuses on the theory of each method and includes examples** that can be calculated without a computer, providing a complete understanding of the procedures. Methods include the analytic hierarchy process (AHP), ANP, simple additive weighting method, ELECTRE, PROMETHEE, the gray relational model, fuzzy integral technique, rough sets, and the structural model. Integrating theory and practice; **the second part of the book illustrates how methods can be used to solve real-world MADM problems**.

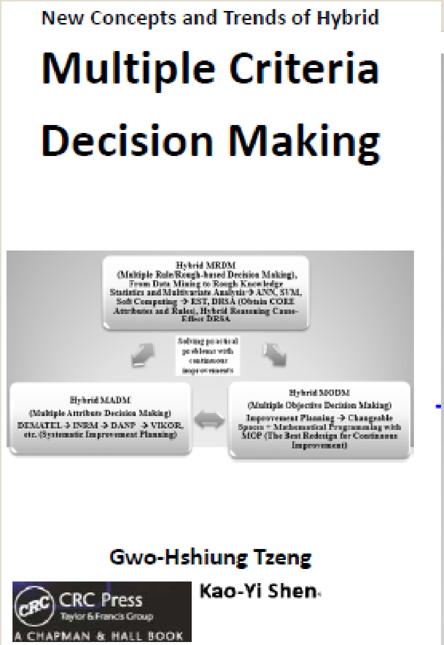


Two New Books (2)

Fuzzy Multiple Objective Decision Making

Gwo-Hshiung Tzeng, Jih-Jeng Huang, CRC Press, Taylor & Francis Group, 2013, 313 pages

Multi-objective programming (MOP) can simultaneously optimize multi-objectives in mathematical programming models, but the optimization of multi-objectives triggers the issue of **Pareto solutions** and complicates the derived answers. To address these problems, researchers often incorporate the concepts of fuzzy sets and evolutionary algorithms into MOP models. Focusing on the methodologies and applications of this field, **Fuzzy Multiple Objective Decision Making** presents mathematical tools for complex decision making. **The first part of the book introduces the most popular methods used to calculate the solution of MOP in the field of multiple objective decision making (MODM). The authors describe multi-objective evolutionary algorithms; expand de novo programming to changeable spaces, such as decision and objective spaces**; and cover network data envelopment analysis. **The second part focuses on various applications**, giving readers a practical, in-depth understanding of MODM. A follow-up to the authors' *Multiple Attribute Decision Making: Methods and Applications*, this book guides practitioners in using MODM methods to make effective decisions. It also extends students' knowledge of the methods and provides researchers with the foundation to publish papers in operations research and management science journals.

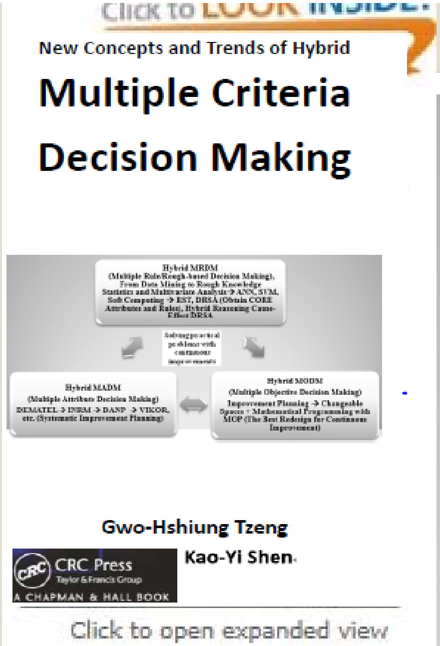


Will Be Published New Books (1/3)

New Concepts and Trends of Hybrid Multiple Criteria Decision Making

Gwo-Hshiung Tzeng, Kao-Yi Shen, CRC Press, Taylor & Francis Group, 2016

New concepts and trends of **hybrid MCDM** for Tomorrow into three main categories, namely, **Multiple Rule/Rough-based Decision Making** (**MRDM**), **Multiple Attribute Decision Making** (**MADM**), and **Multiple Objective Decision Making** (**MODM**) for real-life in solving-problem applications. **Chapter 1 Introduction**: Profile of Traditional MCDM Techniques/Methods, Statistics vs. MCDM approaches, History of MADM, History of MODM, Developments of Computational Machine and Soft Computing for Decision Aids, Basic Concepts of Fuzzy Sets, Basic Notions of Rough Sets, Emerging Trend in Multiple Rough/Rule-Based Decision-Making (MRDM), Outline of the Book. **Part One (NEW CONCEPTS AND TRENDS OF HYBRID MCDM)**, **Chapter 2 New Hybrid MCDM Models for Tomorrow**: Problem-Solving in Traditional MCDM, Why Do We Need New MCDM Approaches, Framework of the New Hybrid MCDM Models for Tomorrow. **Chapter 3 DEMATEL Technique**: The Original DEMATEL Technique, Infeasibility of DEMATEL Technique, Revised DEMATEL Technique, Generalization of DEMATEL Technique, Example. **Chapter 4 DEMATEL Technique for Constructing INRM and Determining DANP**: Methodology for Solving the Real World Problems, Constructing influential network relation map (INRM), Determining influential weights by using DEMATEL-based ANP (DANP), Hybrid Dynamic Multiple Attribute Decision Making (HDMADM).



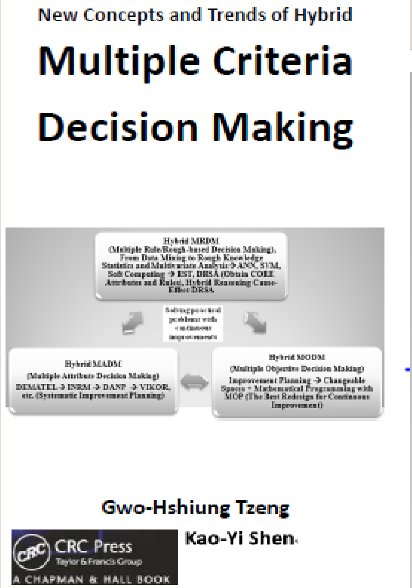
Will Be Published New Books (2/3)

New Concepts and Trends of Hybrid Multiple Criteria Decision Making

Gwo-Hshiung Tzeng, Kao-Yi Shen, CRC Press, Taylor & Francis Group, 2016

Chapter 5 Traditional MADM and New Hybrid Modified MADM: Traditional MADM for Ranking and Selection (AHP and ANP in relatively important weights, Using Max-Min Approach in Normalization for Performance Integration, Evaluation Methods for Performance Improvement (Additive and Non-Additive Type Aggregators). **Chapter 6 New Thinking with Changeable Spaces for hybrid MODM:** Essential Ideas of MODM, Pareto Solution in Traditional MODM, Changeable Spaces with the Idea of Aspiration Levels, The Future of MODM. **Chapter 7 Hybrid Multiple Rule/Rough-based Decision Making (MRDM):** Basic Concept of Rule-Based Approach, DRSA and VC-DRSA for Rule/Rough-Based Knowledge, Formal Concept Analysis (FCA) for Implication Rules, DEMATEL-Based Directional Flow Graph (DFG), Hybrid Approach for Improvement Planning. **Part two (EMPIRICAL CASES),**

- **Chapter 8 (The case of DEMATEL analysis for assessing information risk),**
- **Chapter 9 (A new hybrid MADM model combining DANP with VIKOR for improving**
- **e-store business),**
- **Chapter 10 (A hybrid MADM approach for improving the performance of green**
- **suppliers in the TFT-LCD industry),**



Will Be Published New Books (3/3)

New Concepts and Trends of Hybrid Multiple Criteria Decision Making

Gwo-Hshiung Tzeng, Kao-Yi Shen, CRC Press, Taylor & Francis Group, 2016

- **Chapter 11** (Exploring smart phone improvements based on a hybrid MADM model),

Chapter 12 (Evaluating the implementation of business-to-business m-commerce by SMEs based on a new hybrid MADM model)

Chapter 13 (DANP with VIKOR for selecting glamor stocks)

Chapter 14 (De Novo planning for strategic alliance)

Chapter 15 (The example of changeable space in MODM)

Chapter 16 (VC-DRSA with DEMATEL for the semiconductor industry)

Chapter 17 (DRSA+DANP+VIKOR for evaluating commercial banks)

Chapter 18 (VC-DRSA with FCA-based DANP improvement planning for the IT industry)

- **Chapter 19** (A fuzzy integral-based model for supplier evaluation and improvement)
- **Chapter 20** (DRSA+DANP with Fuzzy Integral for life insurers on improvement
- lanning)

Will Be Published New Books

Mining for Data, Text and Web – Theory and Application Perspectives of Big Data

<u>Jih-Jeng Huang, Gwo-Hshiung Tzeng</u>. With the popular of the big data issue, the aims of the book are to provide big data mining methods and their applications in the real world. The main themes of the proposed book include data mining, text analytics, web mining, and distributed data mining algorithms. The objective is to provide the needed analytic skills for a qualitied data scientist. This book will also present some newly developed methods, including social network analysis, distributed data mining, massively parallel processing, etc. **in Multiple Rule/Rough-based Decision Making** (**MRDM**). **In data process/mining, DRSA with multi-criteria was developed clause-effect flow graph if-then rules based on DEMATEL in combining new hybrid MADM model, called MRDM (Multiple Rule/Rough-based Decision Making)**

A new concepts and trends of combined/hybrid MCDM approach for improving performance planning

Hybrid MRDM
(Multiple Rule/Rough-based Decision Making), From Data Mining to Rough Knowledge Statistics and Multivariate Analysis→ ANN, SVM, Soft Computing → RST, DRSA (Obtain CORE Attributes and Rules), Hybrid Reasoning Cause-Effect DRSA

Solving practical problems with continuous improvements

Hybrid MADM
(Multiple Attribute Decision Making)
DEMATEL→ INRM → DANP → Modified VIKOR, etc. (Systematic Improvement Planning)

Hybrid MODM
(Multiple Objective Decision Making)
Improvement Planning → Changeable Spaces + Mathematical Programming with MOP (The Best Redesign for Continuous Improvement)

New Concepts and Trend of Hybrid Multiple Criteria Decision Making (1/5)

- We have developed and created some new research methods, "New Concepts and Trend of Hybrid Multiple Criteria Decision Making" (will be published in This Year by CRC Press, Taylor & Francis) for Problems-Solving in Real World Situations to enhance the ability/competency for solving the real world problems via thinking and reasoning by logic in doing the real works and researches.

Dear Prof. Gwo-Hshiung Tzeng and Dr. Kao-Yi Shen,

We are pleased to inform you that, after academic reviews by researchers in different continents, and after our business consideration, your title ***New Concepts and Trends of Hybrid Multiple Criteria Decision Making*** was approved to be published by CRC Press, Taylor & Francis Group, catalogue assigned is **K29759**.

We warmly welcome you to join our authors' community. Different teams in CRC Press will work with you from pre-submission to sales of your and book globally.

In one month, the ISBNs will be generated for printed version and E-book version respectively, and a project coordinator will be assigned to assist you on manuscript preparation and submission.

Thank you very much for choosing CRC Press, we'll try our best to ensure the quality and to maximize the exposure or your book.

Sincerely yours

Commissioning Editor:

Ruijun He

1/04/2016.

New Concepts and Trend of Hybrid Multiple Criteria Decision Making (2/5)

- These research methods for solving the practical problems/issues are, including the interrelated three parts:
  (1)Hybrid MRDM (Multiple Rule/Rough-based Decision Making),
  (2) Hybrid MADM (Multiple Attribute Decision Making), and
  (3) New MODM (Multiple Objective Decision Making) based on Changeable Spaces.

New Concepts and Trend of Hybrid Multiple Criteria Decision Making (3/5)

- These research methods "New Concepts and Trends of Hybrid MCDM for Solving the Real World Problems", can be effectively used in various industry and its various sectors (public and private sectors to promote the overall thinking by systematics for Industry-Academia Collaboration in all areas for solving the practical problems, and fulfil to enrich the best achievements "aspiration level". Then these best achievements can all present the empirical case-study of Taiwan as examples; the results not only can be applied to solve real Innovation/Creativity of the problem-improved towards for reaching "aspiration level" in the real cases, but also can be rewritten and published in SSCI/SCI high-cited well-known international journals.

New Concepts and Trend of Hybrid Multiple Criteria Decision Making (4/5)

- New research features and contributions of these research methods "New Concepts and Trends of Hybrid Dynamic MCDM Research Methods" in results are follows:

  In Hybrid MRDM,

  (1) how can be easy to understand and control from **"Big-Data" to extract the "CORE Attribute" for** decision-makers in making decision through/ combining MADM and MODM;

  In Hybrid MADM ((2)-(5) items),

  (2) how can find the influence relation matrix in the key "aspects and criteria" to establish the Influential Network Relation Map (**INRM**) and find the **influential weights of DANP** (DEMATEL-based ANP) in dependence and feedback problems by **cause-effect via interrelationship** in the practical situations;".

  (3) how fulfill "**problem-solving and improvement**" for the overall consideration "aspects and criteria" can be **all towards for reaching "aspiration level";**

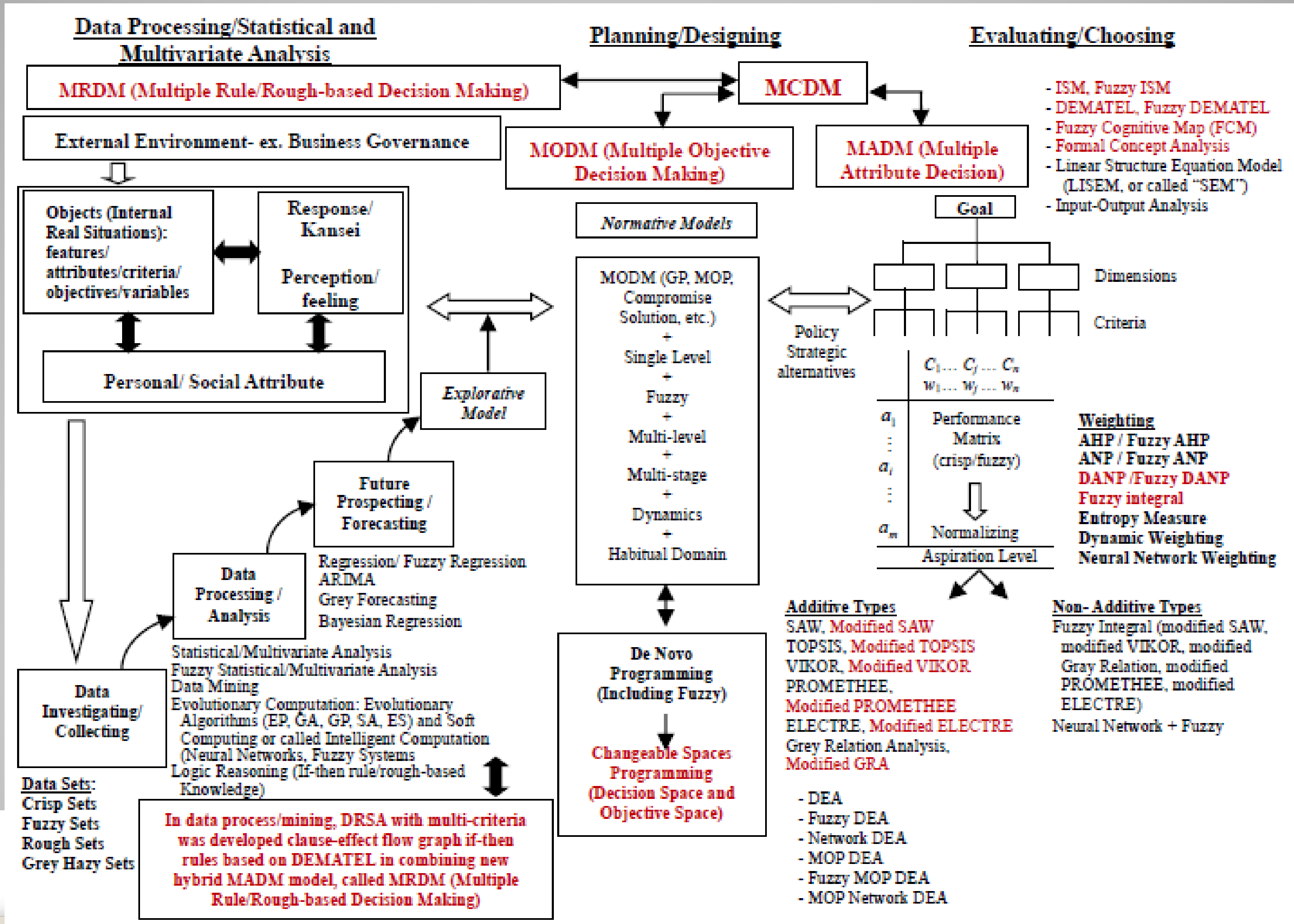
New Concepts and Trend of Hybrid Multiple Criteria Decision Making (5/5)

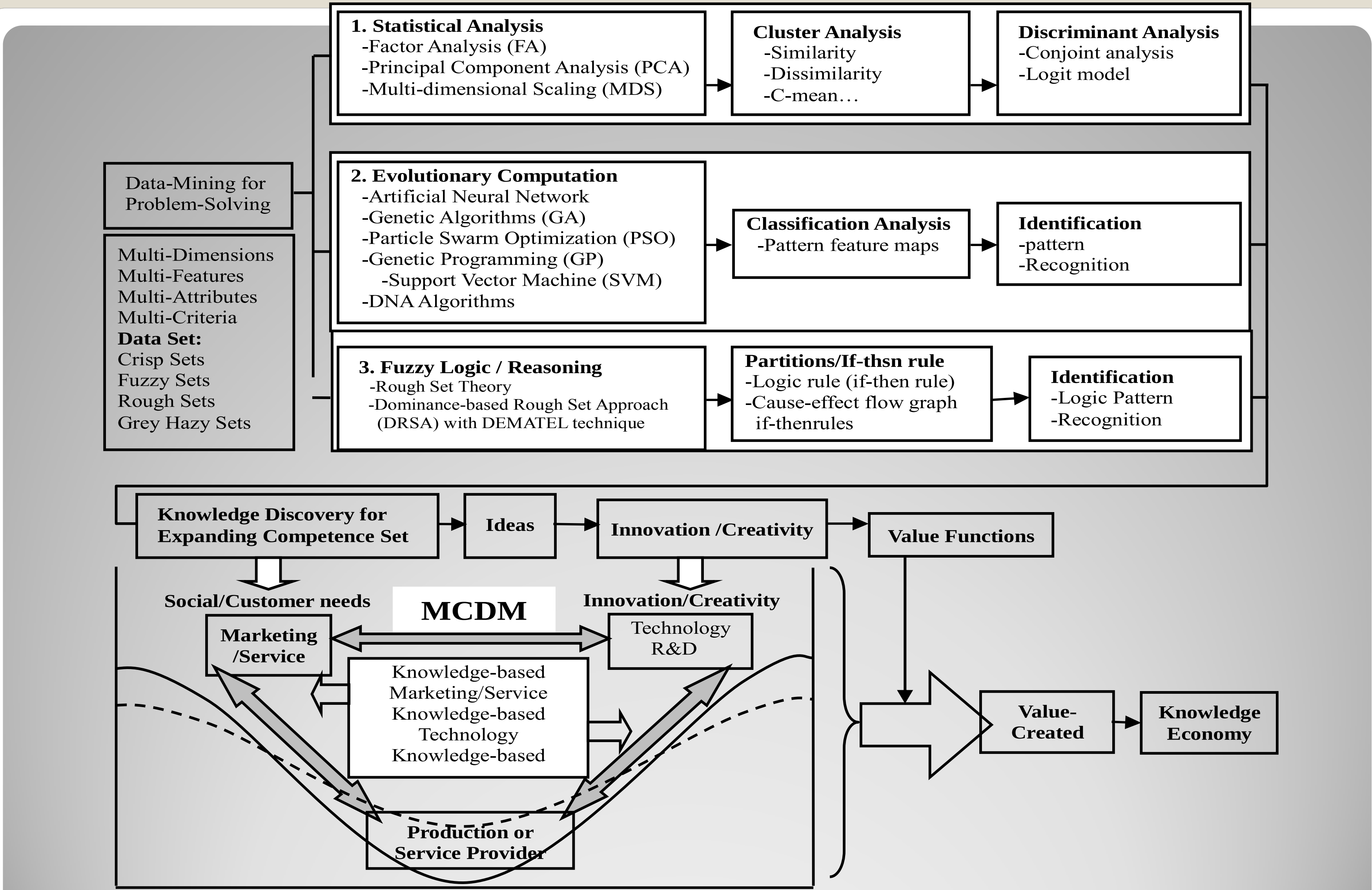
(4) how can **build the integrity (overall-view) of improvement strategies by systematics based on INRM**;

(5) how can solve the "multiple attribute in assessment to integrate integrate each criterion into each aspect (dimension) and overall as a **"non-additive type"** (or called "**super-additive type**") in the real world situations; in **Hybrid MODM**,

(6) how to **break-through resource constraints** in the past cannot be changed in multi-objective **programming problem (**in traditional "Mathematical Programming"), can be towards for **"aspiration level" to pursue "objectives**-achieving" by using "**changeable spaces (decision space and objective space) programing.**

Basic Concepts of Course Systems in "Research Methods" for Problem-Solving

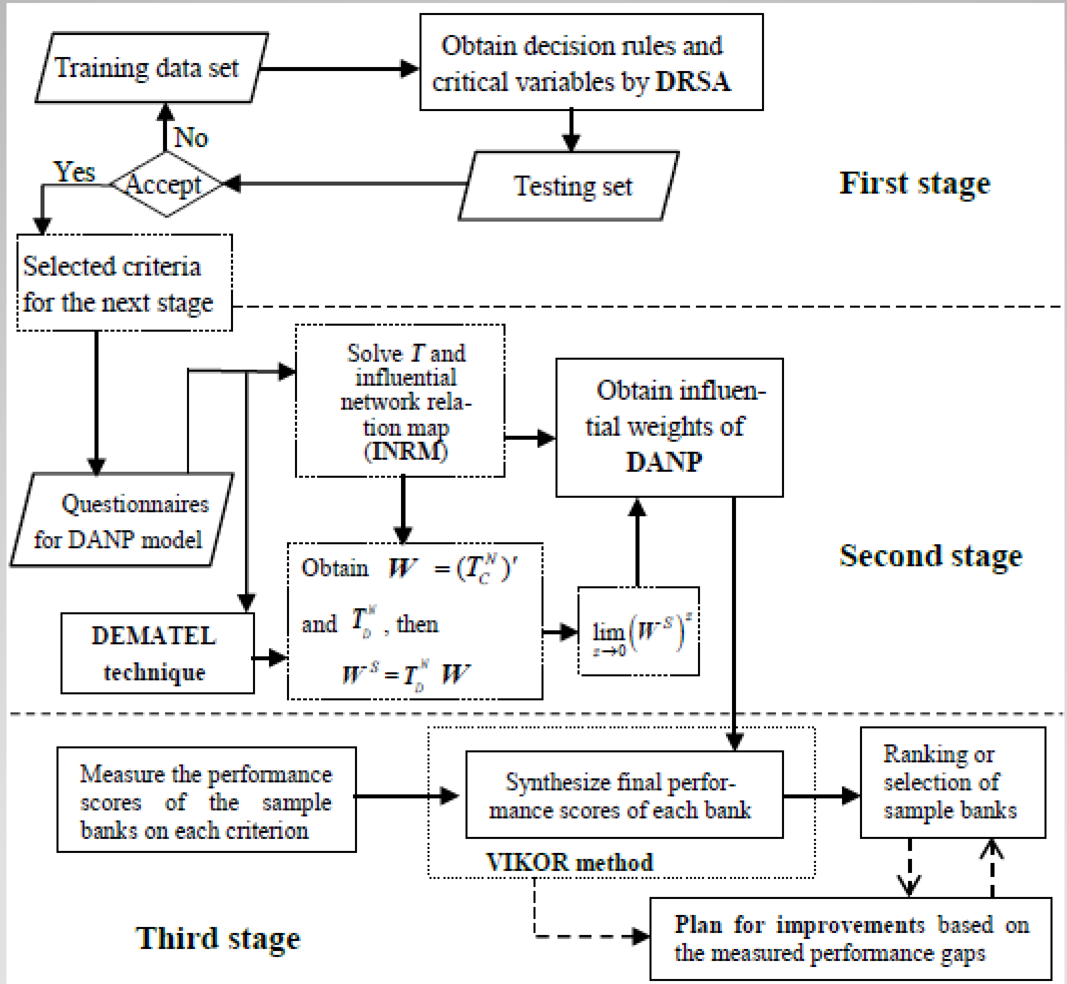




Data Mining Concepts of Intelligent Computation for Knowledge Economy

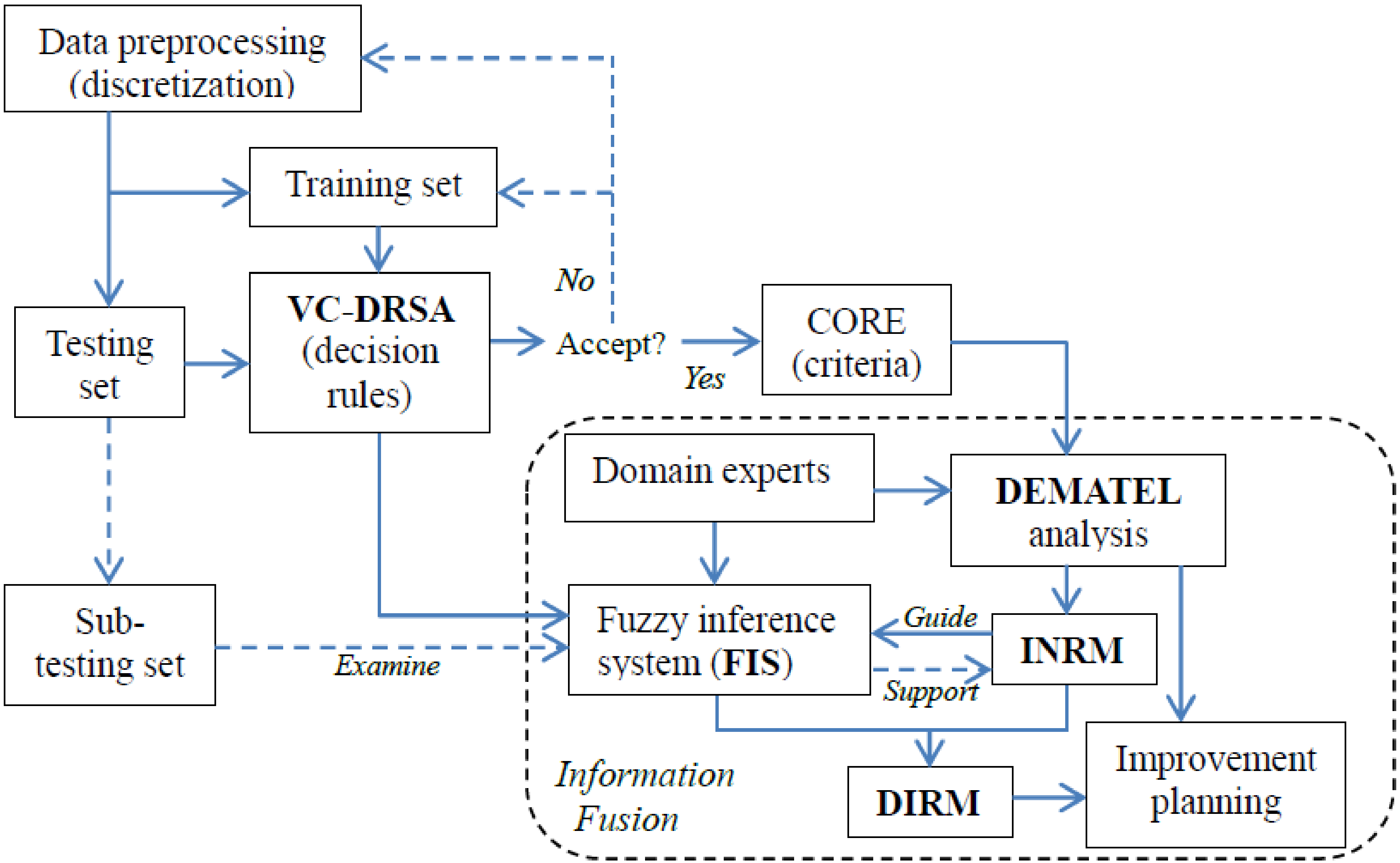
(Tzeng and Huang, 2011; Liou and Tzeng, 2012; Peng and Tzeng, 2013)

MRDM: The illustration of the infused methods for the proposed model



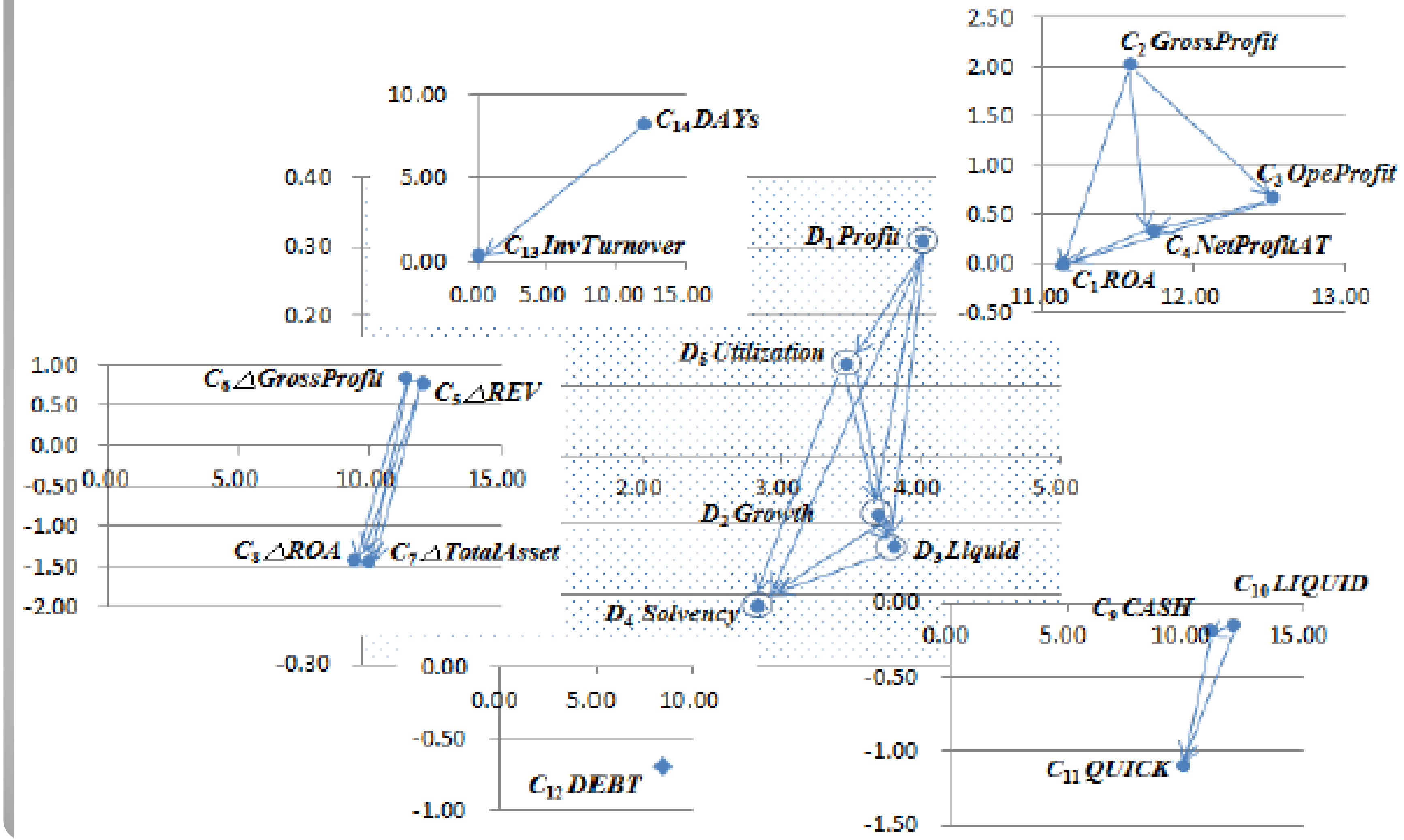
Shen, K.Y. and Tzeng, G.H. (2015). A decision rule-based soft computing model for supporting financial performance improvement of the banking industry, Soft Computing, 19(4), 859-874.

MRDM: Framework of Research Flow



Shen, K.Y., Tzeng, G.H. (2015). A new approach and insightful financial diagnoses for the IT industry based on a hybrid MADM model, *Knowledge-Based Systems*, Volume 85, September 2015, Pages 112–130(SCI, 2014, IF: 3.058)

MRDM: Influential network relation map for guiding influence directions



Shen, K.Y., Tzeng, G.H. (2015). A new approach and insightful financial diagnoses for the IT industry based on a hybrid MADM model, *Knowledge-Based Systems*, Accepted on 2015-04-20 (SCI, 2014, IF: 3.058), Forthcoming

MRDM: Directional flow graph of the strongest "at most Bad" decision rule

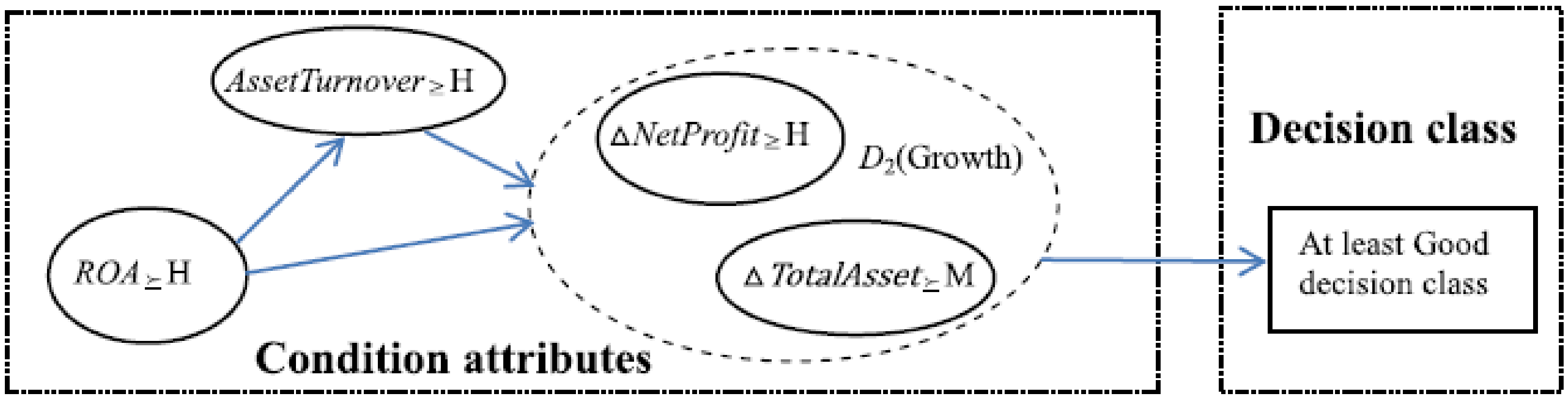
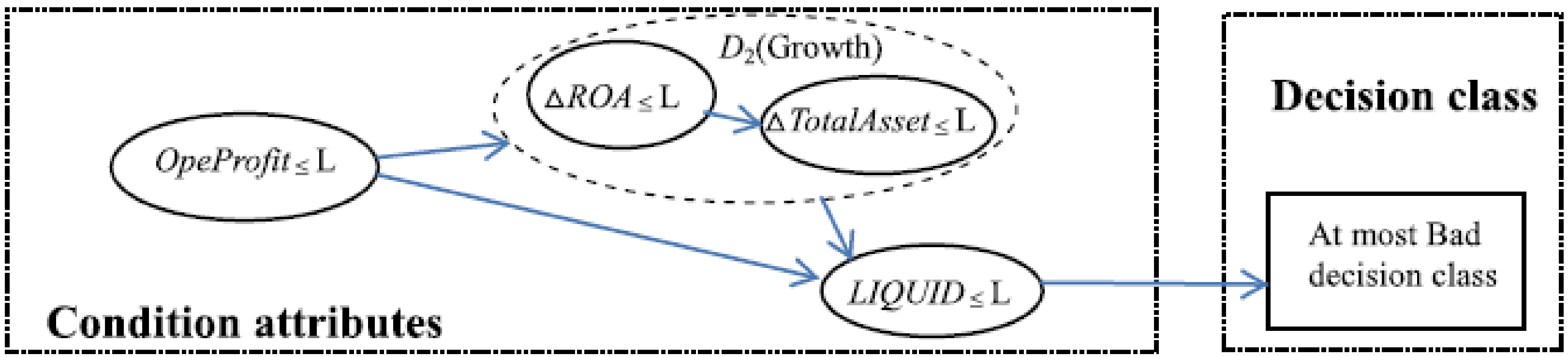
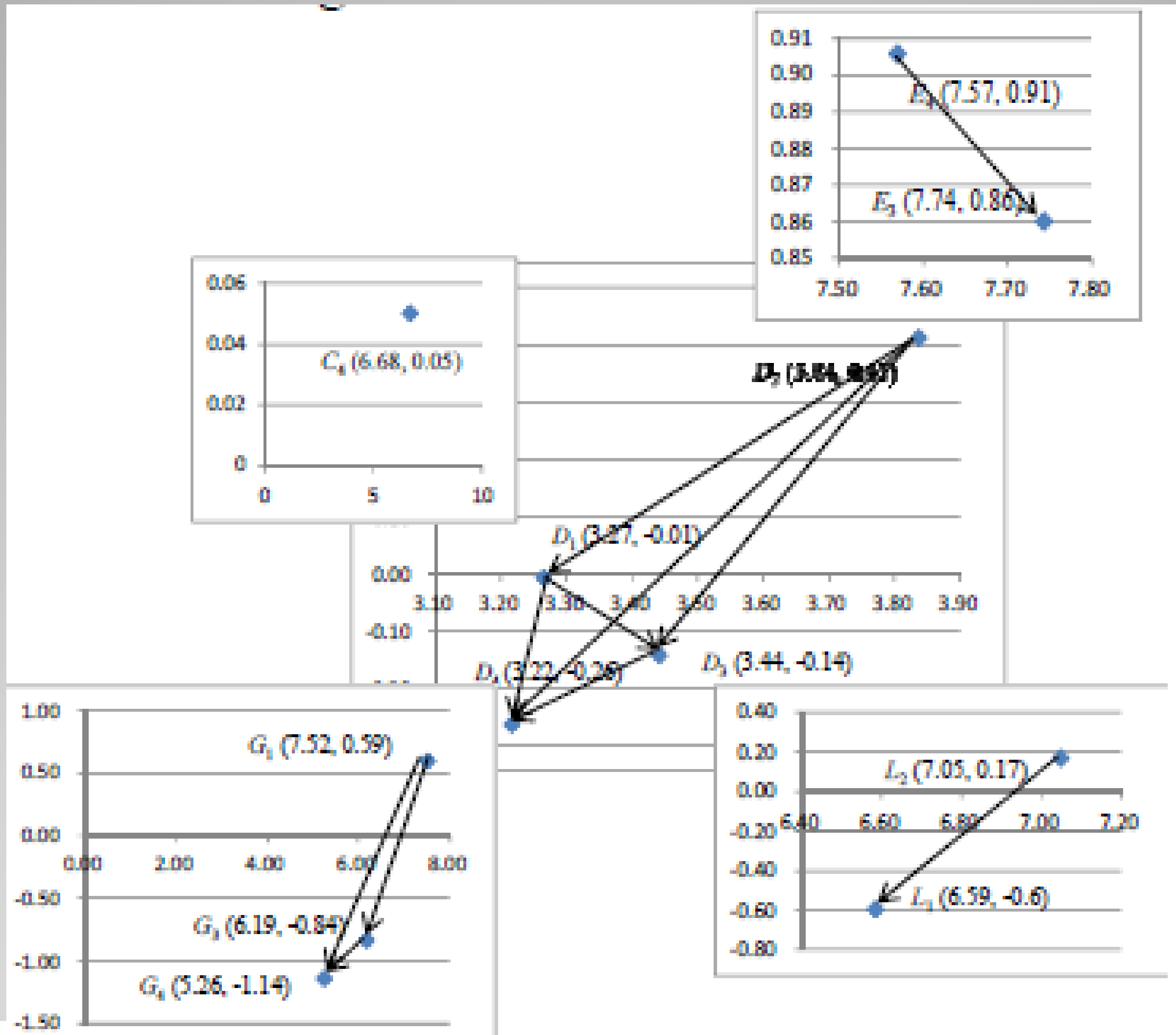


Fig. 5a. Directional flow graph of the strongest "at least Good" decision rule.



Shen, K.Y., Tzeng, G.H. (2015). A new approach and insightful financial diagnoses for the IT industry based on a hybrid MADM model, *Knowledge-Based Systems*, Accepted on 2015-04-20 (SCI, 2014, IF: 3.058), Forthcoming

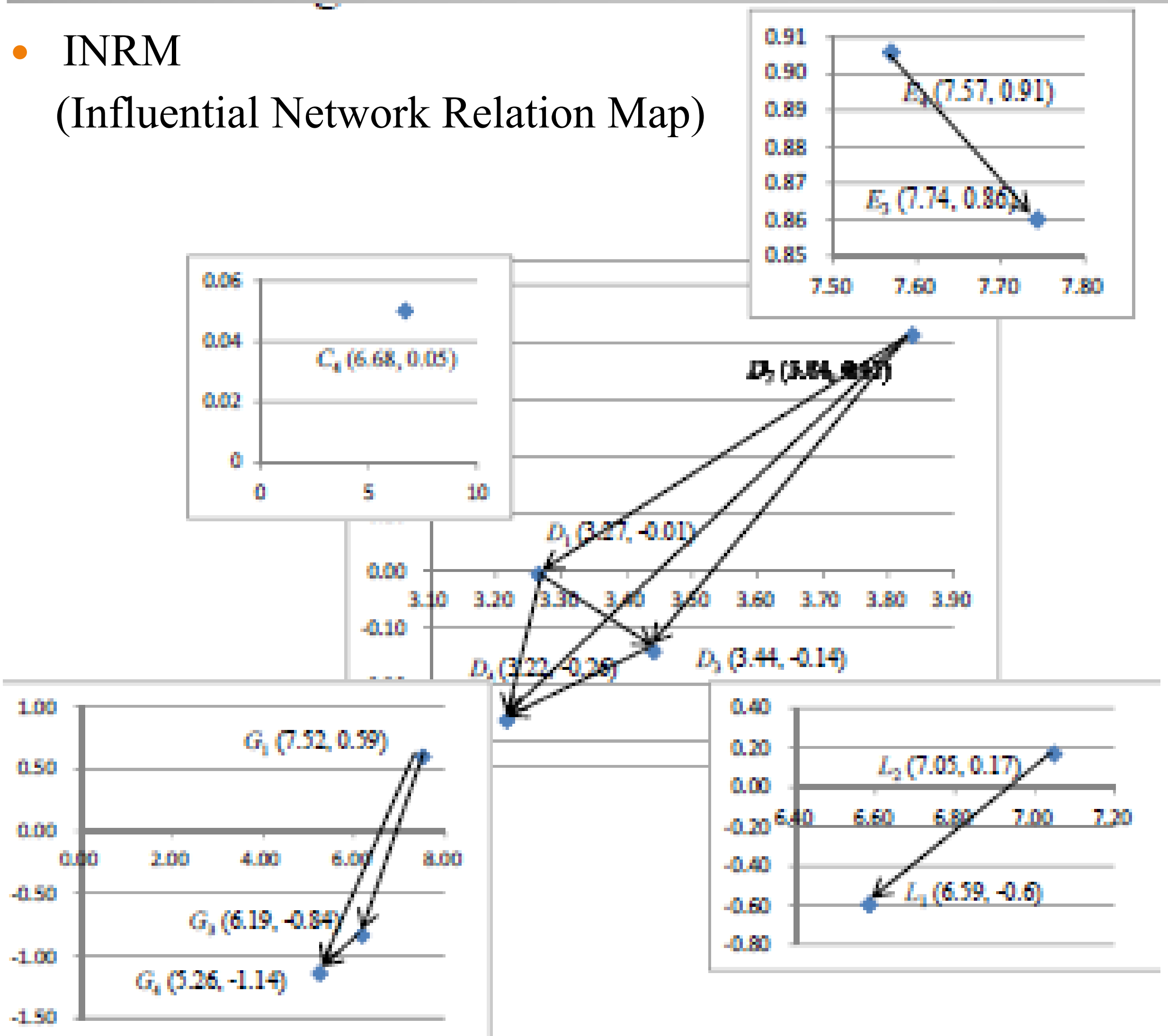
MRDM: Internetwork relationship map



Shen, K.Y., Tzeng, G.H. (2015). A new approach and insightful financial diagnoses for the IT industry based on a hybrid MADM model, *Knowledge-Based Systems*, Accepted on 2015-04-20 (SCI, 2014, IF: 3.058), Forthcoming

MRDM: Internetwork relationship map

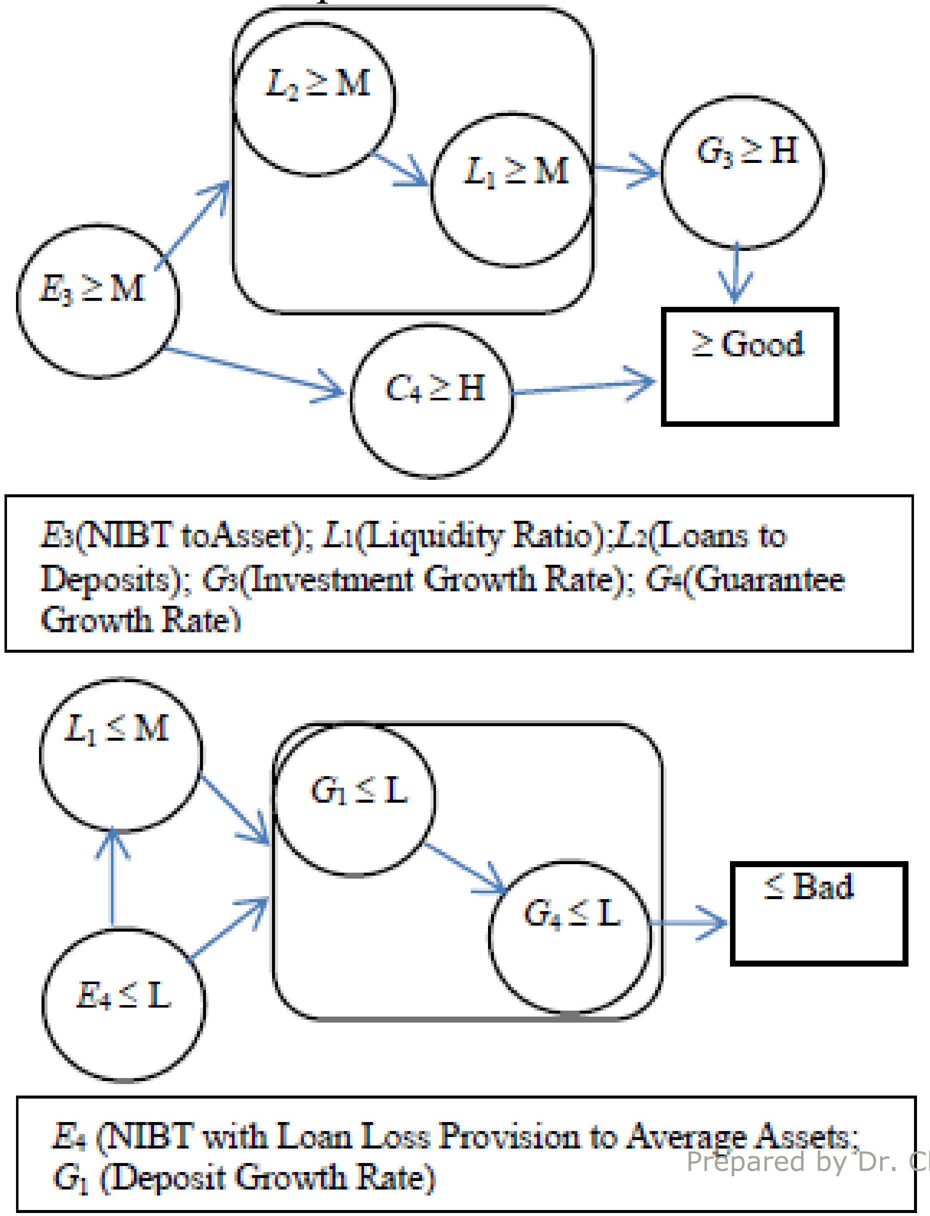
- INRM
  (Influential Network Relation Map)



Shen, K.Y. and **Tzeng, G.H. (2014).** DRSA-based Neuro-Fuzzy Inference Systems for the Financial Performance Prediction of Commercial Bank. ***International Journal of Fuzzy Systems***, **16(2), 173-183.**

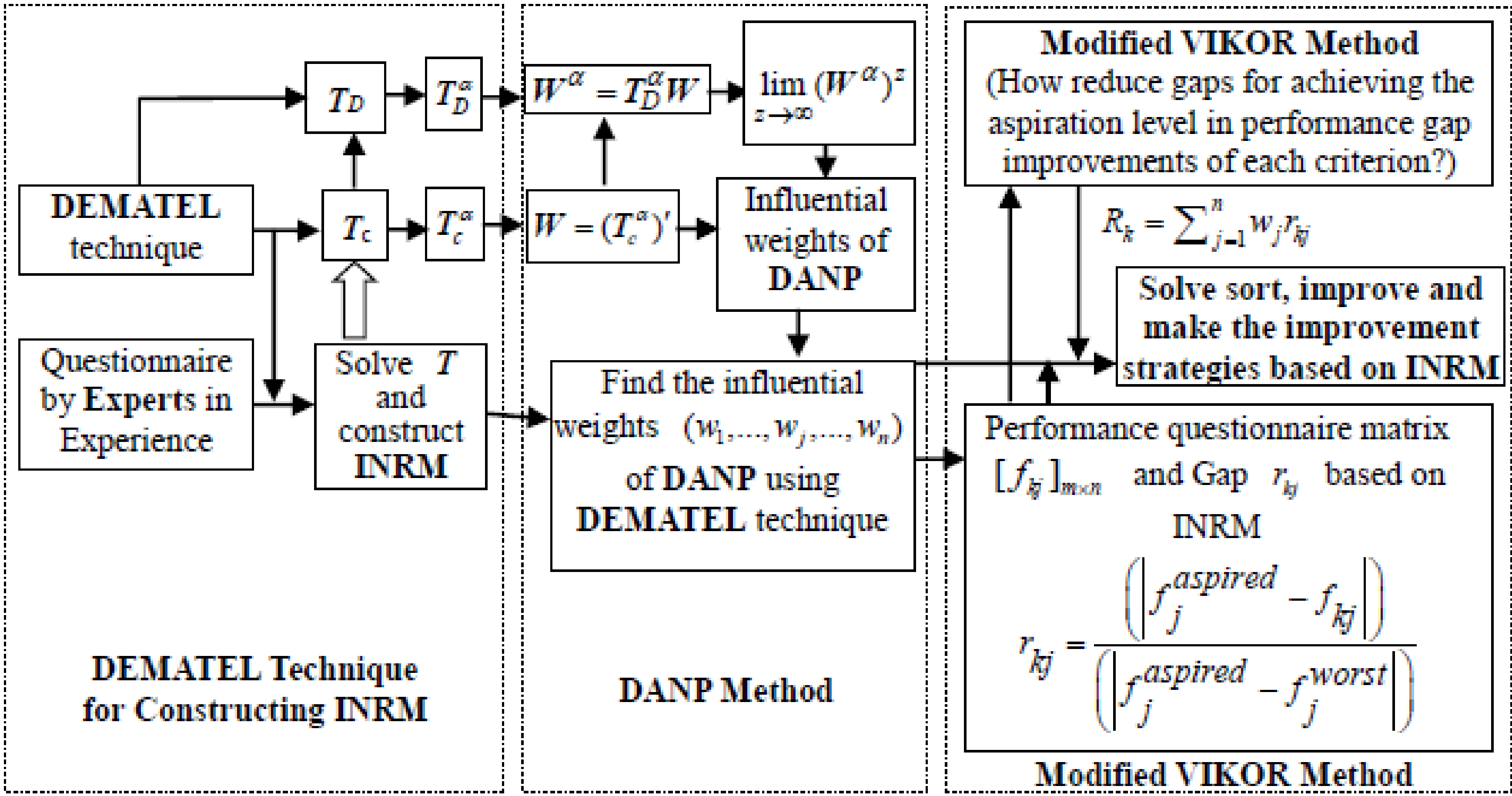
MRDM: Directional flow graph (DFG).

Flow Graph on Cause-Effect



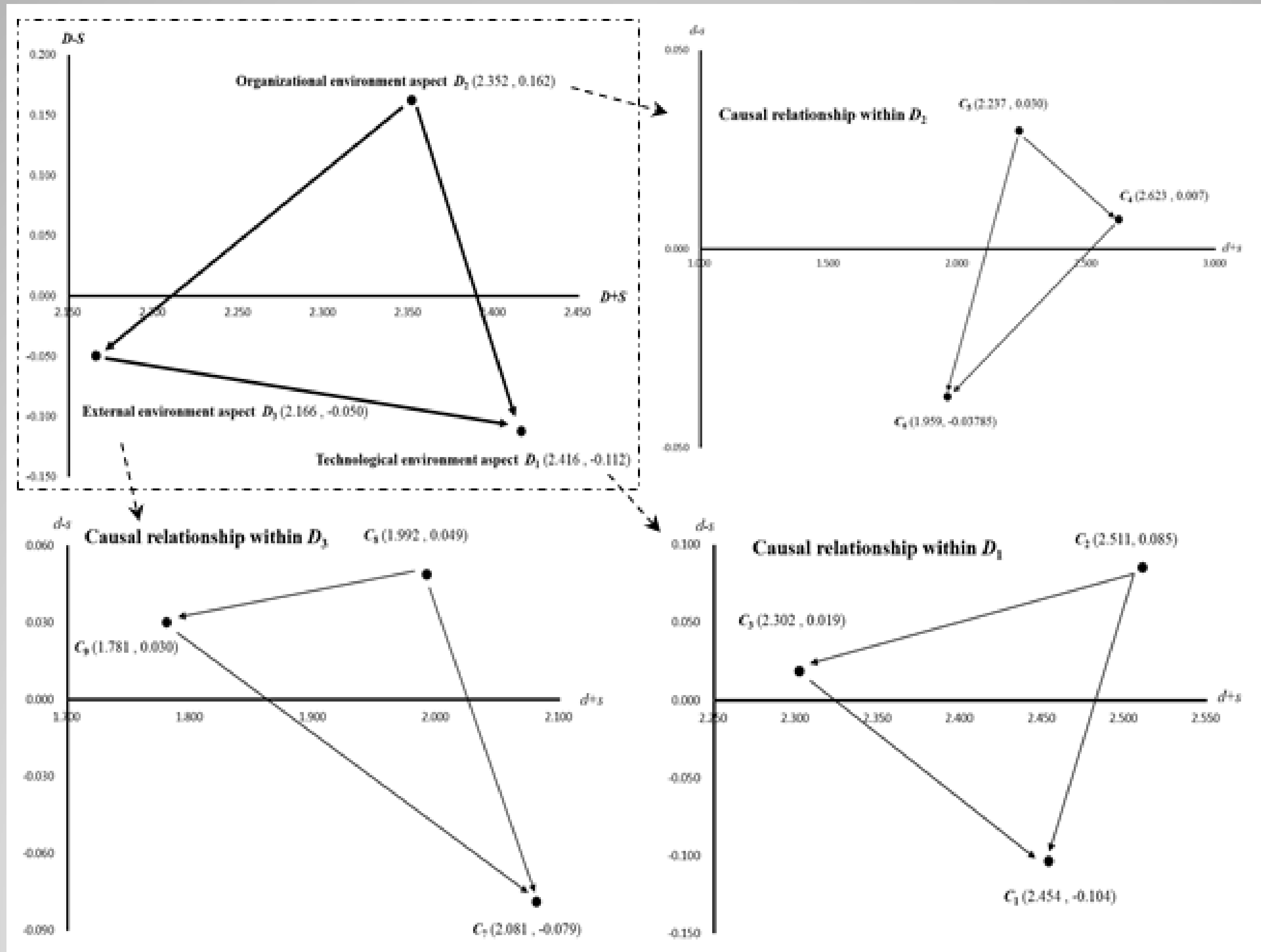
Shen, K.Y. and **Tzeng, G.H. (2014).** DRSA-based Neuro-Fuzzy Inference Systems for the Financial Performance Prediction of Commercial Bank. ***International Journal of Fuzzy Systems***, **16(2), 173-183.**

MADM: The Basic Concepts of New Hybrid MADM Model in the Real World

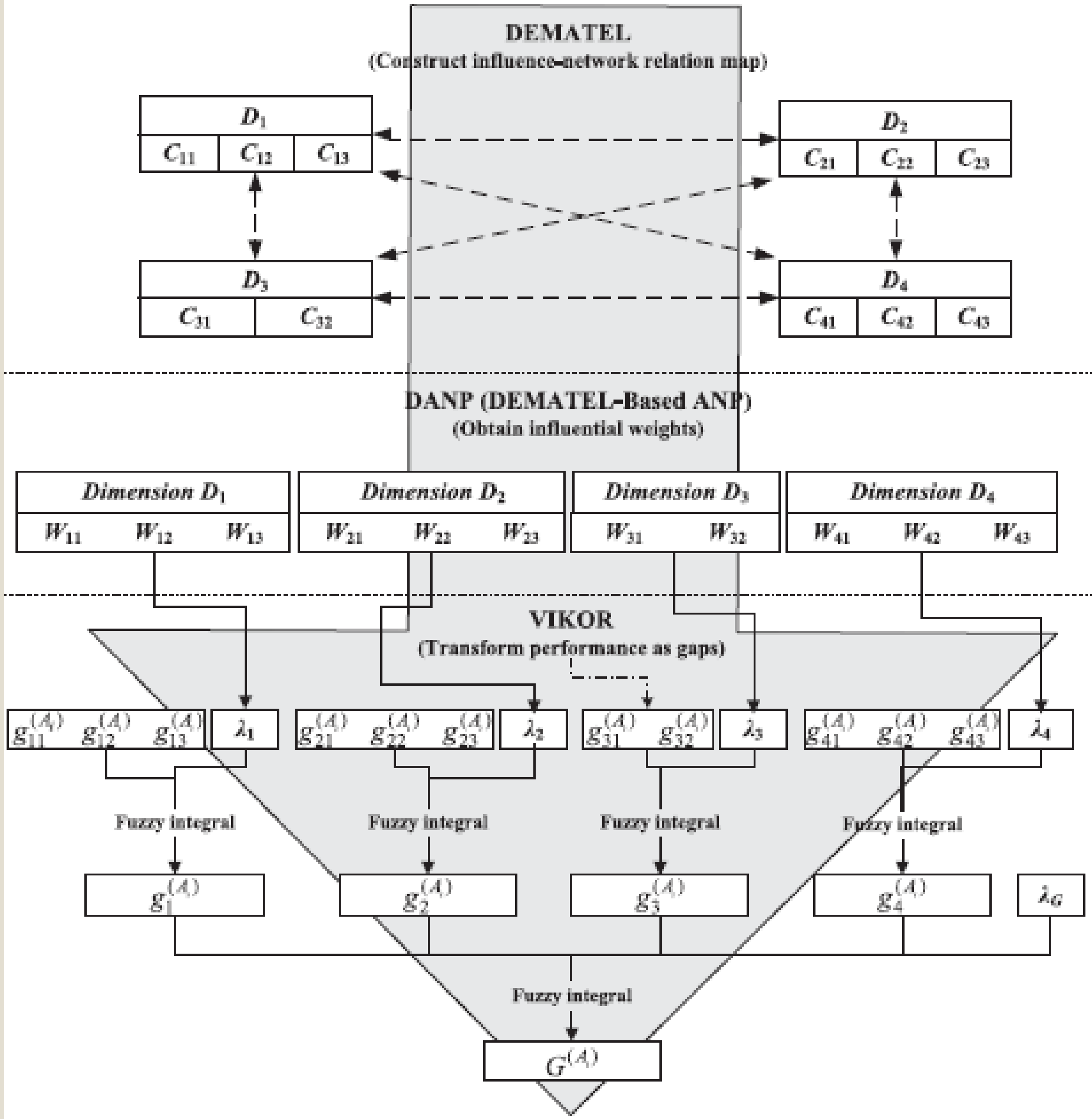


Lu, M.T., Hu, S.K., Huang, L.H., Tzeng, G.H. (2015). Evaluating the implementation of business-to-business m-commerce by SMEs based on a new hybrid MADM model, Management Decision, 53(2), 290 - 317

MADM: The influential network relationship map (INRM) of each aspect and criterion



Lu, M.T., Hu, S.K., Huang, L.H., Tzeng, G.H. (2015). Evaluating the implementation of business-to-business m-commerce by SMEs based on a new hybrid MADM model, Management Decision, 53(2), 290 - 317

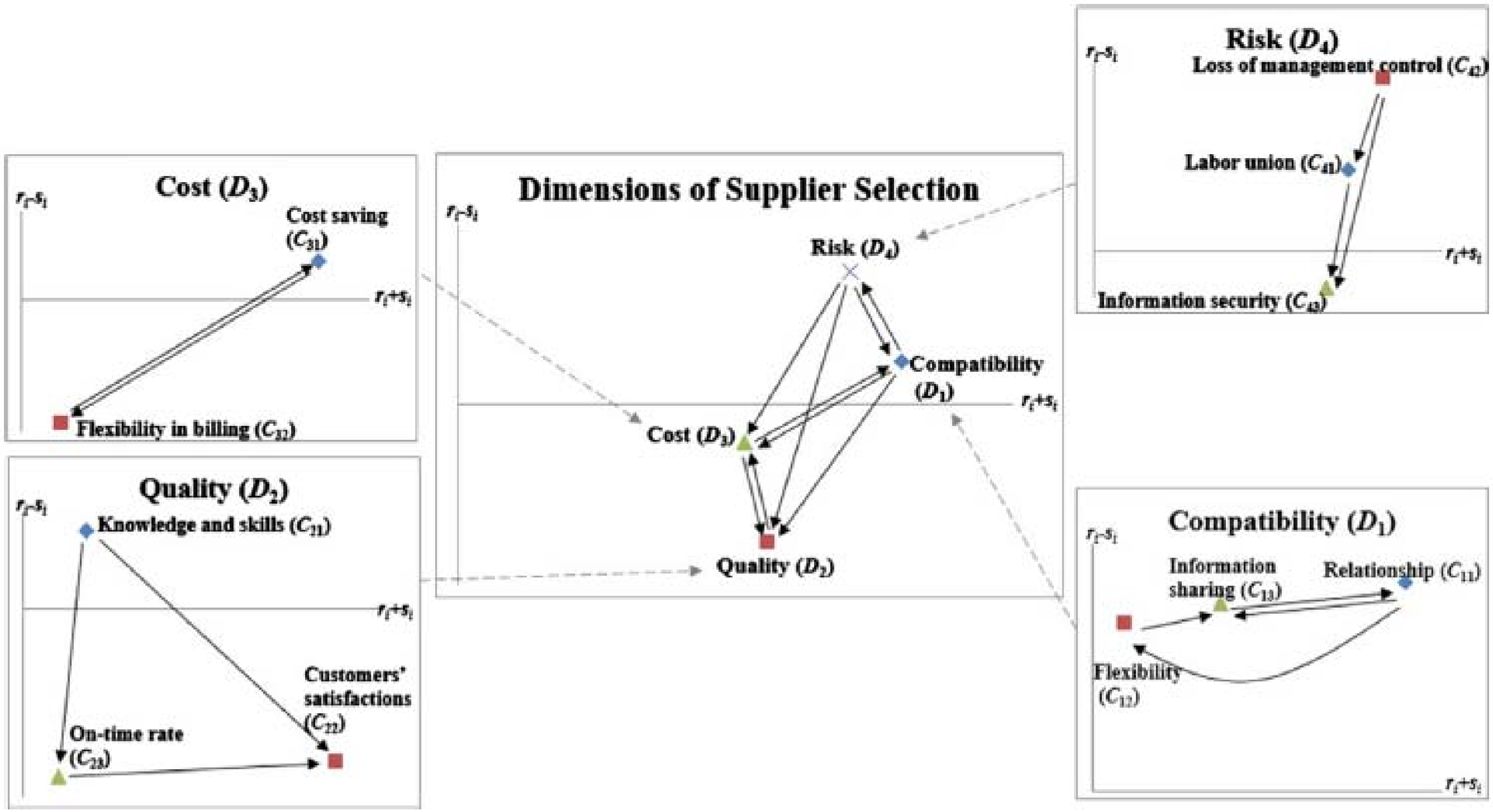


- **MADM: New Concepts and Trends of MADM model in Real World Problem for Tomorrow**

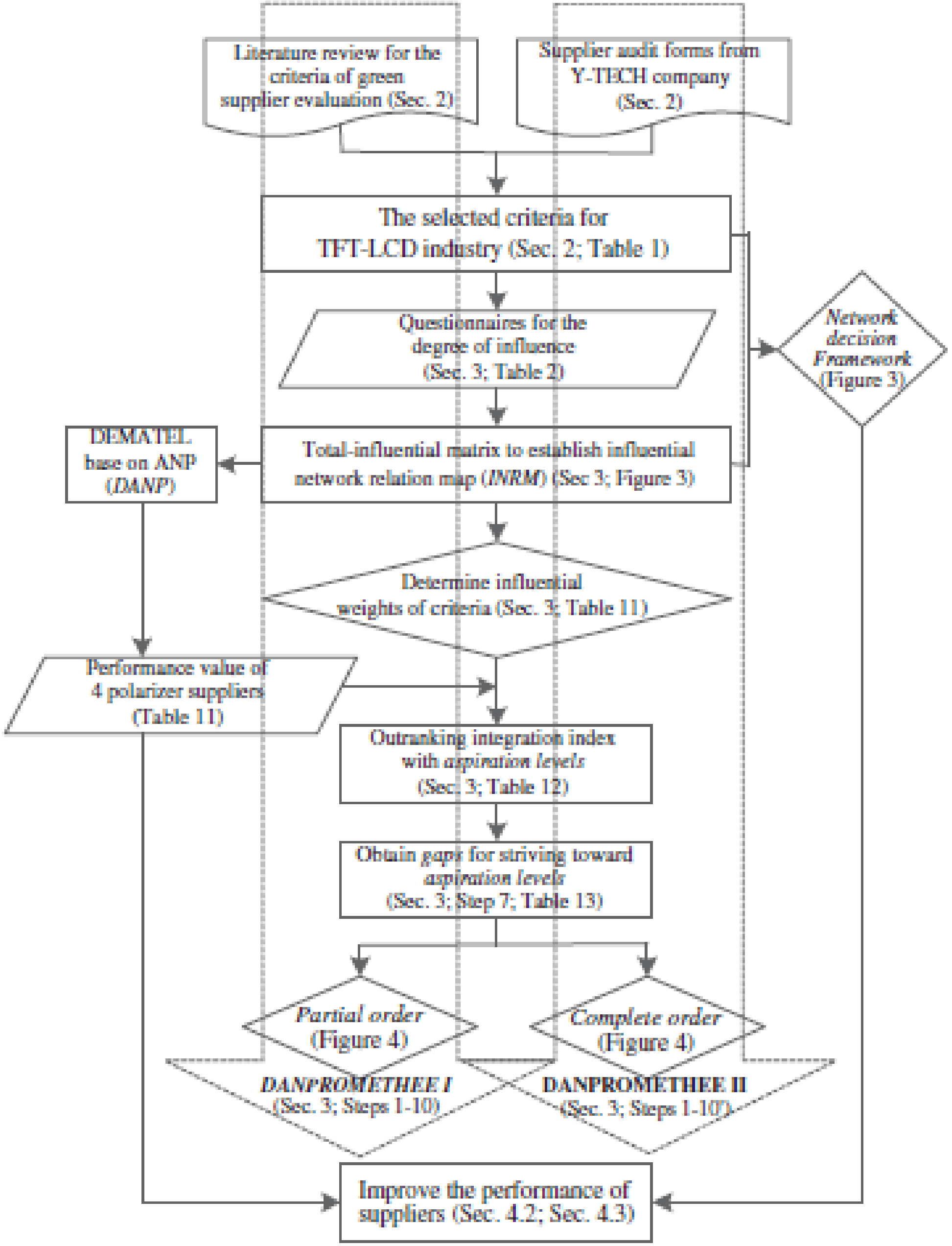
- **DEMATEL for building INRM and finding influential weights of DANP → Modified VIKOR based on INRM by using super-additive (or called non-additive) form for performance improvement.**

Liou, James J.H., Chuang, Y.C., Tzeng, G.H. (2014). A fuzzy integral-based model for supplier evaluation and improvement. *Information Sciences*, Volume 266. 10 May 2014, Pages 199–217 (SCI, Impact factor: 3.643, 5-Year Impact Factor: 3.676, 2012; SNIP: 3.425, 2013).

MADM: The influential network relationship map (INRM) of each aspect and criterion



Liou, James J.H., Chuang, Y.C., Tzeng, G.H. (2014). A fuzzy integral-based model for supplier evaluation and improvement. *Information Sciences*, Volume 266. 10 May 2014, Pages 199–217 (SCI, Impact factor: 3.643, 5-Year Impact Factor: 3.676, 2012; SNIP: 3.425, 2013).

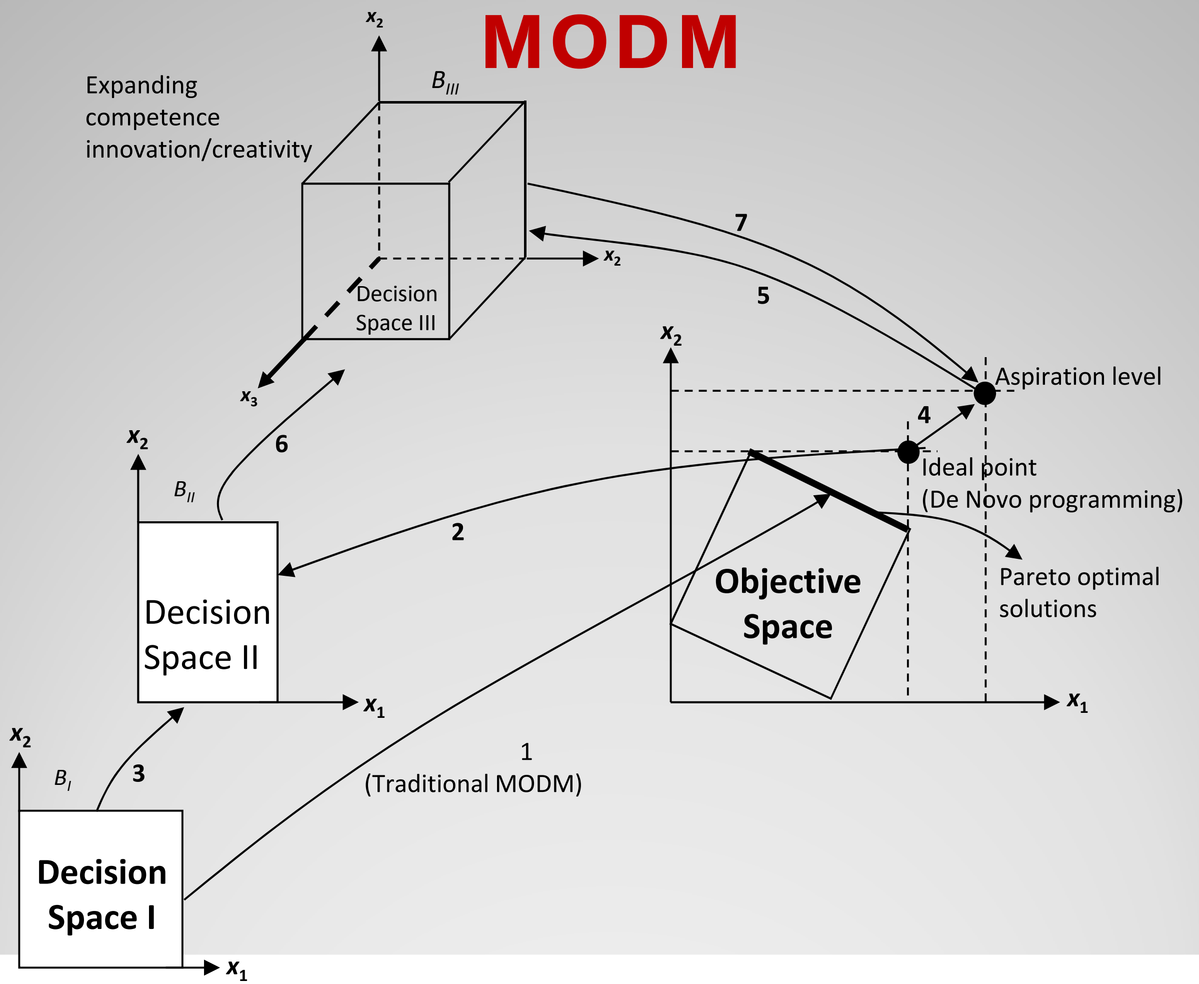


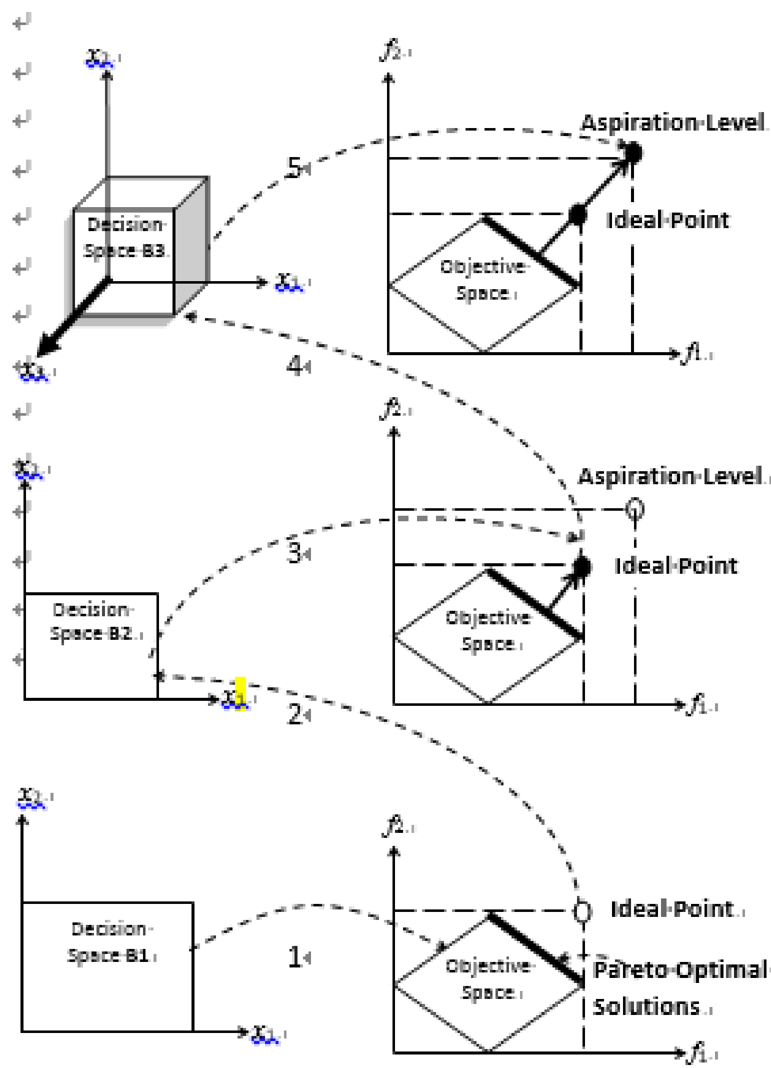
- **MADM: New Concepts and Trends of MADM model in Real World Problem for Tomorrow**

- **DEMATEL for building INRM and finding influential weights of DANP → Modified Grey Relation Analysis (Modified GRA) based on INRM**

- **Liou, James J.H., Tamosaitiene, J., Zavadskas, E., Tzeng, G.H. (2015). A new hybrid COPRAS-G MADM model for improving and selecting suppliers in green supply chain management, *International Journal of Production Research*, Acceptance of revised paper ID TPRS-2014-IJPR-1730.R1. (SCI/SSCI, 2013 Impact Factor: 1.323)**

MODM: The new concepts and Trends of changeable spaces and aspiration level in MODM





Concept	Graphical Representation	Approach
Value (Win-Win)		making aspired decisions by expanding competence sets through innovation
		Making Ideal decisions through re-allocating limited resources
Price (Win-Lose)		Making Pareto optimal decisions through traditional MOP methods

MODM: Changeable spaces programming by stages in new thinking of MODM for aspiration level

Huang, J.J., Tzeng, G.H. (2014), New thinking of multi-objective programming with changeable spaces – In search of excellence, *Technological and Economic Development of Economy*, 20(2): 242–261 (SSCI, IF: 5.605, 2011; IF: 3.235, 2012).

Tzeng, G.H., Huang, K.W., Lin, C.W., BJC Yuan, B.J.C. (2014). New idea of Multi-objective programming with changeable space for improving the unmanned factory planning, *Management of Engineering & Technology (PICMET)*, 2014 Portland International Conference on Pages: 564-570.

New Publications
(Important papers)

- **Liou, James J.H. and Tzeng, G.H. (Corresponding author**) (**2012**), Comments on "Multiple criteria decision making (MCDM) methods in economics: An overview", ***Technological and Economic Development of Economy***, **18(4)**, 672-695 **(SSCI, IF: 5.605, 2011; IF: 3.235, 2012).** Z Turskis, EK Zavadskas (VGTU)
- **Kua-Hsin Peng, Gwo-Hshiung Tzeng** (**Corresponding author**) (**2013**), A hybrid dynamic MADM model for problems-improvement in economics and business, ***Technological and Economic Development of Economy***, **19(4)**, 638–660 (**SSCI, IF: 5.605, 2011; IF: 3.235, 2012).**
- **James J.H. Liou, Yen-Ching Chuang, Gwo-Hshiung Tzeng** (**Corresponding author**) **(2013),** "A fuzzy integral-based model for supplier evaluation and improvement, ***Information Sciences*, 266**, 199–217 (Impact factor: **3.643** (5-Year) Impact Factor: **3.676 (2012), IPP: 4.686 (2013)**).
- **Jih-Jeng Huang, Gwo-Hshiung Tzeng** (**2014**), New thinking of multi-objective programming with changeable space - In search of excellence, ***Technological and Economic Development of Economy***, **20(2), 254-273 (SSCI, IF: 5.605, 2011; IF: 3.235, 2012).**
- **James J.H. Liou (2013),** New concepts and trends of MCDM for tomorrow – in honor of Professor Gwo-Hshiung Tzeng on the occasion of his 70th birthday, ***Technological and Economic Development of Economy***, **19(2)**, 367–375 (**SSCI, IF: 5.605, 2011; IF: 3.235, 2012).**

Publications in International Books (English)

- **G. H. Tzeng, H. F. Wang, U. P. Wen, P. L. Yu (Editors). Multiple Criteria Decision Making**, Springer-Verlag, 1994.
- **Gwo-Hshiung Tzeng, Jih-Jeng Huang (2012). Multiple Attribute Decision Making: Methods and Applications, CRC Press, Taylor & Francis Group, 2011, 349 pages.**
- **Gwo-Hshiung Tzeng, Jih-Jeng Huang (2013). Fuzzy Multiple Objective Decision Making, CRC Press, Taylor & Francis Group, 2013, 313 pages**
- **Gwo-Hshiung Tzeng, Kao-Yi Shen (2016). New Concepts and Trends of Hybrid Multiple Criteria Decision Making, CRC Press, Taylor & Francis Group, In Press.**

New Journal Papers

2016

- Kuan-Wei Huang, Jen-Hung Huang, **Gwo-Hshiung Tzeng** (**Corresponding author**) (**2016**). New Hybrid MADM Model for Improving Competence Sets: Enhancing a Company's Core Competitiveness, ***Sustainability*** **2016**, *8*(2), 175; doi: 10.3390/su8020175, **SSCI**, **Impact Factor: 0.942 (2014), 1.077 (2013)**
- Liou, James J.H.; Tamosaitiene, Jolanta; Zavadskas, Edmundas; **Tzeng, Gwo-Hshiung (2016)**. A new hybrid COPRAS-G MADM model for improving and selecting suppliers in green supply chain management, ***International Journal of Production Research***, **54(1),** 114-134. **DOI:** 10.1080/00207543.2015.1010747. (**SCI/SSCI**, **2014 Impact Factor: 1.477**)

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- **Fu-Hsiang Chen, Gwo-Hshiung Tzeng** (2015). Probing Organization Performance Using a new Hybrid Dynamic MCDM Method Based on the Balanced Scorecard Approach, ***Journal of Testing and Evaluation*, 43(4): 1-14** (**SSCI**, **IF: 0.279**, 2014).
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Talk: New Concepts and Trends of Hybrid MCDM Model for Tomorrow

- **New concepts and trends of hybrid MCDM model for Tomorrow**
- **How consider for solving the real world**
- **Basic concepts of ideas and thinking in trends**
- **New concepts and trends of hybrid MCDM model for Tomorrow: Some examples for the real cases**
  - MRDM: Dominance-based rough set approach (DRSA) MCDM
  - MADM: DEMATEL, DANP (DEMATEL-based ANP), Integration (Additive: Modified SAW, Modified VIKOR, Modified Grey Relation Analysis, Modified PROMETHEE, Modified ELECTRE; Non-additive: Fuzzy Integral)
  - MODM: Changeable Spaces Programming
- **Conclusions**

New concepts and trends of hybrid MCDM model for Tomorrow

Solving Actual Problems

(Relax and relieve the traditional assumption/hypothesis in unrealistic problems)

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (1/15)

- Which concepts and how trends in future prospects of MCDM field for Tomorrow?
- Which problems will be improved for satisfying the users'/customers'/social needs in real (behavior pattern-rules, marketing, the whole people, etc.) situations?
- How overall considering problems in total, objectives, aspects/dimensions, and criteria can be achieved the aspiration levels?

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (2/15)

Development change of MCDM in daily live

- Decision making (DM) is extremely intuitive when considering single criterion problem, since we only need to choose the alternative with the highest preference rating.
- However, when DM evaluate alternatives with multiple criteria, many problem, such as how weights-measuring of criteria, preference dependence, interrelationship (existing dependence and feedback) among criteria in the real world, and so on, seem to complicate the problems and need to be overcome by more sophisticated methods (Tzeng and Huang, 2011, 2013).
- A typical multiple criteria decision making (MCDM) is a scientific analytical method for evaluating a set of alternatives based on the consideration of multiple criteria to determine a priority ranking and selection for alternative implementation (Tsaur, Tzeng, & Wang., 1997)

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (3/15)

Ranking and section

- **The general objective of MCDM methods is to help the decision maker in selecting the best alternative from the limit number of feasible alternatives under the presence of multiple choice criteria and diverse criteria priorities (Jankowski, 1995; Mollaghasemi and Pet-Edwards, 1997).**
- **MCDM techniques have been used in recent years to solve a wide variety of problems in ranking and selection (Chen and Liao, 2004; Hung and Chiang, 2008; Ou Yang, Shieh, Leu and Tzeng, 2008). Normalized performance matrix by using max-min approach**
- **Additionally, MCDM methods aim at improving the decisions quality, re-consideration how making the decisions process more definite, reasonable and efficient for avoiding "Pick the best apple among a barrel of rotten apples" (避免由「一堆爛蘋果中找出最好的蘋果」).**

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (4/15)

- However, typical MCDM methods are often used to deal with problems in management or solving various problems that is characterized by several non-commensurable and conflicting (competing) criteria, and there may be no solution that satisfies all criteria simultaneously (Ou Yang et al., 2011).
- Using the normalized performance matrix by using **max-min** additive evaluation approach to eliminate non-commensurable and conflicting (competing) in traditional approach adopting (**select and rank the best apple among a barrel of rotten apples)**

Why and how toward new concepts and trends of MCDM approach?

- Additionally, conventional MCDM only consider the crisp/fuzzy decision problems, based on the additive concept along with the independence assumption, however, it should be highlighted that the criteria are usually interactive in practical MCDM problem.

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (5/15)

- Furthermore, conventional MCDM only allow us to choose and rank alternatives or strategies, however, **the problem in real world is dynamic and complication, so we need to develop an appropriate hybrid MCDM for evaluating, improving, and choosing the best alternatives/strategies to reduce performance gaps continuously for achieving win-win aspired/desired levels.**
- Therefore, in the **purpose/goals** of our research projects, **in order to overcome the defects of conventional MCDM methods, new hybrid MCDM methods should be developed to solve and improve the complication dynamic problem in real world by dynamic concepts, how can achieve or toward the aspiration level** (Simon incorporated the basic concept of the "aspiration level" in his work, receiving the Nobel Prize in Economics in 1978).

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (6/15)

Main categories of MCDM (traditional and new approaches)

- Hwang and Yoon (1981) classified **MCDM** problems into two main categories, namely multiple attribute decision making (**MADM**) and multiple objective decision making (**MODM**), based on the different purposes and data types.
- **MADM** is able to consider multiple criteria at the same time and helps the decision maker evaluate and estimate the best case based on the characteristics of a limited number of alternative cases for ranking and selection (Tzeng et al., 2002a, 2002b; Opricovic and Tzeng, 2002, 2003, 2007; Tzeng et al., 2005, Lu et al., 2013; Ferreira, et al., 2014) in the traditional approach.
- **MODM** exist particularly in the areas of design/planning, and generally involve attempting to optimize goals by considering the various interactions within the given constrains, so that both decision and objective spaces are changeable in our new research concepts.

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (7/15)

- Even in data process/mining, **DRSA (Dominance-based Rough Set Approach)** with multi-criteria was developed clause-effect flow graph if-then rules based on DEMATEL in combining new hybrid MCDM model which also included this project research.
- Therefore, this study of **new concepts and trends of hybrid dynamic MCDM model** are focused on how integrating three parts of **MRDM** (**Multiple Rule/Rough-based Decision Making**, data process/mining), **MADM (Multiple Attribute Decision Making)**, and **MODM (Multiple Objective Decision Making)**, which not only proposed that the traditional MCDM ignored some important new concepts and trends, and needed some assumptions (limitations/defects) to solve the real-world problems, but also proposed several important new concepts and trends of new hybrid MCDM model for solving actual problems (Liou and Tzeng, 2012; Liou, 2013; Peng and Tzeng, 2013, Lu et al., 2014) in applying to evaluate implementation.

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (8/15)

- Therefore, this study of **new concepts and trends of hybrid dynamic MCDM model** are focused on how integrating three parts of **DRMA** (data process/mining), **MADM**, and **MODM**, which not only proposed that the traditional MCDM ignored some important new concepts and trends, and needed some assumptions (limitations/defects) to **solve the real-world problems**, but also proposed several important new concepts and trends of hybrid MCDM model **for solving actual problems** (Liou and Tzeng, 2012; Liou, 2013; Peng and Tzeng, 2013, Lu et al., 2014) in applying to evaluate implementation.
- **Several important (six-points) new concepts and trends of hybrid MCDM model for solving actual problems.** In our approach, the DEMATEL technique, the clause-effect flow graph if-then rules in DRSA, the influential network relation map (INRM), the influential weights of DANP, and the **modified** SAW, VIKOR, Grey Relation Analysis, PROMETHEE, ELECTRE method are also included as following **purposes/goals to reach in our constructions**.

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (9/15)

- **First,** for avoiding "Statistics and Economics are unrealistic in assumptions/hypotheses
- Our research group using logical thinking and reasoning based on basic concept of "**Rough Set Theory (RST)"** to construct the core attributes in if-then rules" from "Big Data". Furthermore, we can use the "**DRSA (dominance-based rough set approach**) or VC-DRSA" to build "Flow Graph" in "**if-then rule-based**" combining DEMATEL technique to construct the **cause-effect** in "if-then rule/Rough-based decision-making" (called **M**ultiple **R**ule/**R**ough-based **D**ecision **M**aking, **MRDM**) as influential relationship flow, called **MRDM.**
- These results can make decision-makers or users easy to understand and grasp the problems in the **causal-effect relationship** combining **DEMATEL technique**. So we also can combine the **"new hybrid MCDM model**", can more obtain effectively to provide the decision-makers for solving the **real world problems-improving**.

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (10/15)

- **Second,** the traditional model assumes the criteria are independent and hierarchical in structure; the previous studies that mainly rely on statistical models (e.g., regressions and time series models) to examine the relationship based on independence, linear, correlation, etc.
- However, **in real-world problems**, the **interrelationships** between the criteria or aspects (or called dimensions) are usually **interdependent and sometimes even exert feedback effects**;
- So we adopt **DEMATEL method to construct influential network relation map (INRM)** and to find the influential weights of DANP using basic concept of ANP (Saaty, 1996) based on influence relation matrix of DEMATEL technique (Ou Yang et al., 2008, 2013; Peng and Tzeng, 2013; Shen et al., 2014; Hu et al., 2014) for solving the inter-dependence and feedback (interrelationship problems) of criteria (or called attributes) aspects (or called dimensions) in the real world problem to **avoid "unrealistic assumptions in Statistics and Economics".**

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (11/15)

- **Third**, the relative good solution from the existing alternatives is replaced by the **aspiration levels** to fit today's competitive markets; so we modified VIKOR method (Opricovic and Tzeng, 2004, 2007), SAW, Grey Relation Analysis (Chiu et al., 2014, Liou et al., 2015), PROMETHEE (Tsui et sl., 2015), ELECTRE to correct traditional Max-Min as ideal point and negative ideal point into aspiration level and the worst value
- The relatively good solution from existing alternatives based on "**max-min"** as goal/target (benchmark) is replaced by **aspiration level and worst value ("aspired-worst" as benchmark)** for avoiding "Choosing the best among inferior options/alternatives", i.e. **for avoiding "pick the best apple among a barrel of rotten apples".**
- Simon incorporated the basic concept of the "**aspiration level**" in his work, receiving the **Nobel Prize** in Economics in 1978.

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (12/15)

- **HA Simon** - Decision and organization, 1972 - innovbfa.viabloga.com ... **The Scottish word "satisficing" (=satisfying) has been revived to denote problem solving and decision making that sets an aspiration level,** searches until an alternative is found that **is satisfactory by the aspiration level criterion,** and selects that alternative (Simon (1957), Part IV ... (Simon, 1978, Nobel Prize)
- For example, the performance value of each criterion can be obtained by using questionnaires with a scale ranging from 0 points (complete dissatisfaction/bad) to 10 points (the best satisfaction/good). Then in this case, we can set the aspiration level as $f_j^{aspire} = 10$ and the worst value as $f_j^{worst} = 0, \ j = 1, 2, ..., n$, called "**aspired-worst**" as benchmark. In contrast to the traditional approach, which sets $f_j^* = \max_k f_{kj}$ and $f_j^- = \min_k f_{kj}$, called "**max-min**" as benchmark.

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (13/15)

- **Fourth**, the new hybrid **MADM** analytical tools are not only used in **ranking and selection**, but also can be used in the performance gaps **improvement** among criteria and its corresponding aspects (or dimensions);
- So the emphasis in the field has shifted from ranking and selection when determining the most preferable approaches to performance improvement of existing methods **based on INRM**, because "**we need a systematic approach to problem-solving**; instead of addressing the systems of the problem,
- i.e., we need to identify the sources of the problem in performance **improvement** based on **INRM**, INRM because "we need a systematic approach to problem-solving; instead of addressing the systems of the problem, we, to **avoid "stop-gap piecemeal"**.

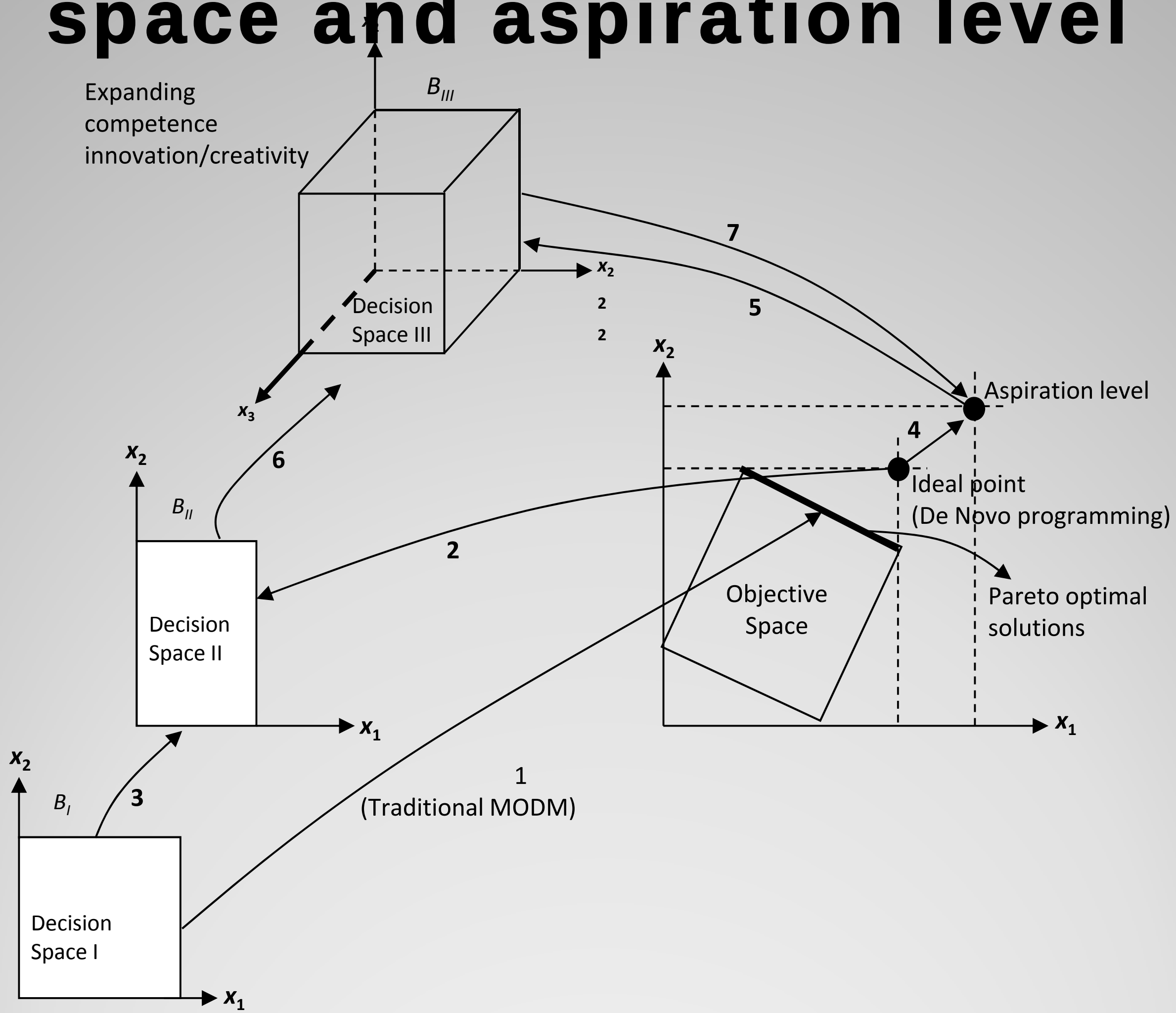
New Concepts and Trends of Hybrid MCDM Model for Tomorrow (14/15)

- **Fifth**, Kahneman and Tversky (Kahneman received the **Nobel Prize** in Economics in 2002), they, in results of many their studies during 1960s, found consumers in products-selecting of multi-attribute preference value are almost different from traditional multi-attribute utility (value-function aggregation in multi-attribute) by using **additive model. , i.e., almost all the results are inconsistent with the real actual problems, when they mistakenly thought preference people have problems**
- Until 1974, Sugeno completed his Doctoral thesis "Theory of fuzzy integrals and its applications" in Tokyo Institute of Technology; fuzzy integrals are, namely, "**non-additive model**" or so-called "**super-additive model**", as a value-function integrated model. So Kahneman based on above basic concept proposed "**Prospect Theory**" in 1978.

New Concepts and Trends of Hybrid MCDM Model for Tomorrow (15/15)

- **Sixth**, based on above points five we can be systematically to find overall thinking the problem-improving for achieving or toward "aspiration levels, the resolve of implementing improvement-strategies in enforcement, how can enforce it?
- **Classical MODM** (Multiple Objectives Decision Making) in thinking of plan/design is based on a fixed set conditions or resources (fixed conditions or resources, this is called "Decision Space", in feasible space to be fixed (i.e., fixed feasible region, this is called "Objective Space") how we can find the Pareto optimal solution?
- We will propose **a new thinking of "MODM models with changeable spaces" to implement and enforce for improvement in solving MADM problems for enhancing the performance values toward achieving the aspiration levels in criteria, dimensions, and overall through innovation and creativity**. This new thinking in changeable spaces programming not only can help decision-makers to **reach win-win planning or design,** but also **can achieve the desired point (aspiration level),** which is better than pursuing the Pareto optimal solutions or ideal point.

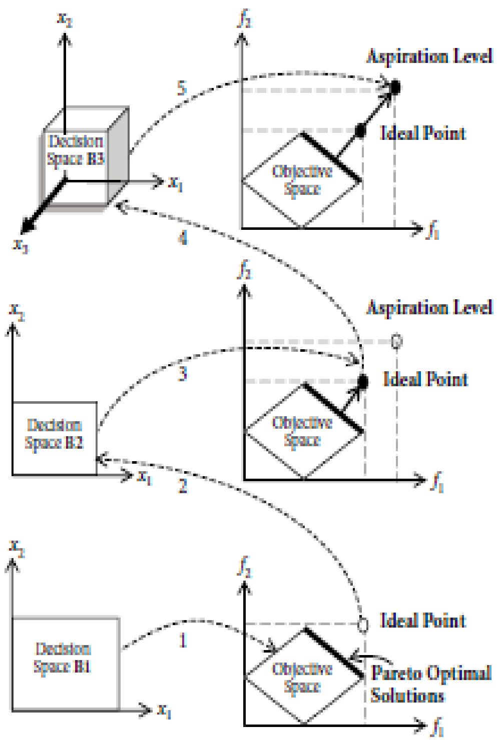
The concepts of changeable decision space and aspiration level



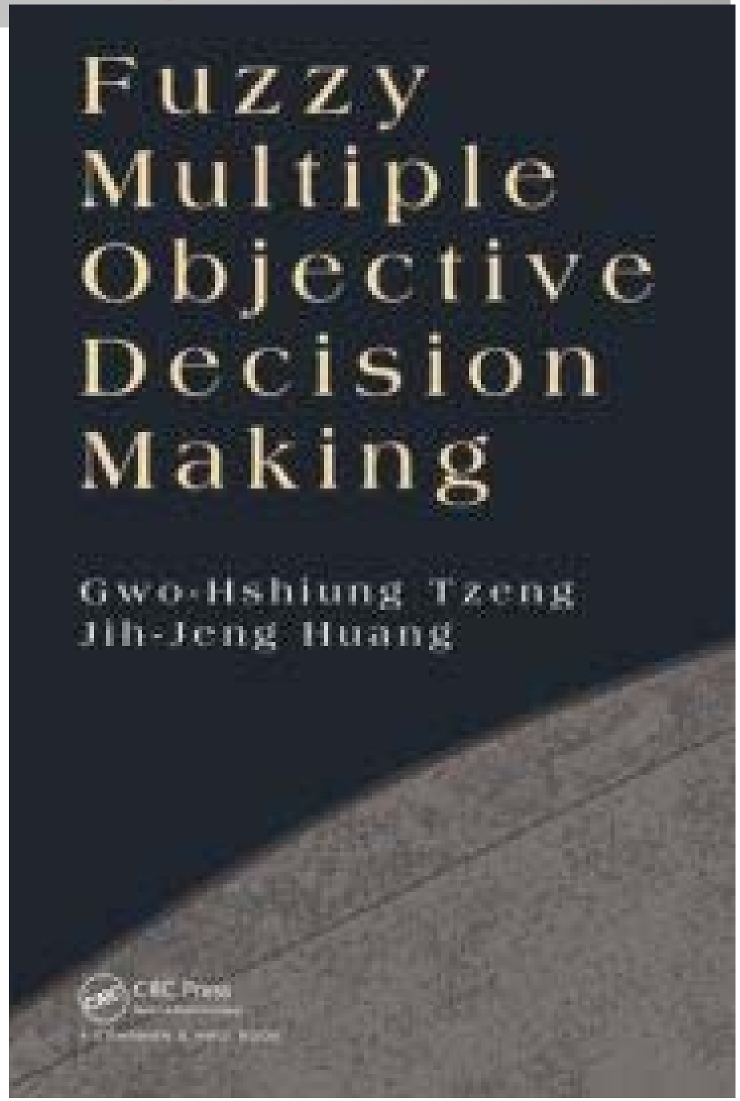
Changeable Space Programming

Toward a MCDM New Era – Professor Tzeng's Roadmap

Philosophy
Taking True Responsibility,
Creating Added Value, and
Making Contribution through MCDM Knowledge to Global Society



concept	Graphical Representation	Approach
Value (Win-Win)		making aspired decisions by expanding competence sets through innovation
		Making Ideal decisions through re-allocating limited resources
Price (Win-Lose)		Making Pareto optimal decisions through traditional MOP methods



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Review

COMMENTS ON "MULTIPLE CRITERIA DECISION MAKING (MCDM) METHODS IN ECONOMICS: AN OVERVIEW"

James J. H. Liou[1], Gwo-Hshiung Tzeng[2]

E-mails: 1jamesjhliou@gmail.com; 2ghtzeng@mail.knu.edu.tw (corresponding author)

Abstract. This paper offers comments on a previously published paper, titled "Multiple criteria decision making (MCDM) methods in economics: an overview," by Zavadskas and Turskis (2011). The paper's authors made great efforts to summarize MCDM methods but may have failed to consider several important new concepts and trends in the MCDM field for solving actual problems. First, the traditional model assumes the criteria are independently and hierarchically structured; however, in reality, problems are often characterized by interdependent criteria and dimensions and may even exhibit feedback-like effects. Second, relatively good solutions from the existing alternatives are replaced by aspiration levels to fit today's competitive markets. Third, the emphasis in the field has shifted from ranking and selection when determining the most preferable approaches to performance improvement of existing methods. Fourth, information fusion techniques, including the fuzzy integral method, have been developed to aggregate the performances. Finally, the original fixed resources in multi-objective programming are divided such that both decision and objective spaces are changeable. In this paper, we add new concepts and provide comments that could be thought of as an attempt to complete the original paper.

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Invited review

NEW CONCEPTS AND TRENDS OF MCDM FOR TOMORROW – IN HONOR OF PROFESSOR GWO-HSHIUNG TZENG ON THE OCCASION OF HIS 70th BIRTHDAY

James J. H. LIOU

Department of Industrial Engineering and Management, National Taipei University of Technology, No. 1, Section 3, Chung-Hsiao East Road, Taipei, Taiwan

Abstract. This article introduces several new concepts and trends in multiple criteria decision making (MCDM) for solving actual problems, as proposed by Professor Gwo-Hshiung Tzeng. These new concepts are as follows: (1) interdependency in real-world problems; (2) replacing the relative good solution from the existing alternatives using aspiration levels; (3) shifting from ranking and selection to performance improvement; (4) information fusion/aggregation; and (5) changeable decision spaces. To honor Prof. Tzeng's contribution in the MCDM field and to commemorate his 70th birthday, this article also highlights his research career in MCDM and some publication list in the past 10 years.

Keywords: MCDM, MADM, MODM, DEMATEL, DANP, VIKOR, Changeable space, Aspiration level.

Reference to this paper should be made as follows: Liou, J. J. H. 2013. New concepts and trends of MCDM for tomorrow – in honor of Professor Gwo-Hshiung Tzeng on the occasion of his 70th birthday, *Technological and Economic Development of Economy* 19(2): 367–375.

1. New trends and concepts in MCDM

Over the past two decades, the development of information technology (IT) has been characterized by a series of positive, but temporary, shocks. The alternate perspective is that IT in Internet communication has produced a fundamental change in the world, leading to a permanent improvement in fast growth-change prospects such as telephone, telegraph, Internet, smart phone, i-phone, i-pad, cloud computing, business, economy, society, and government. What are the prospects for future trends? Which problems will be solved regarding user/customer/societal needs in marketing situations, and how can overall problems in dimensions and criteria be resolved using aspiration levels? The traditional MCDM field ignored some important new concepts and trends and needed several assumptions to solve real-world problems. Therefore, Prof. Tzeng proposed some new concepts for facing tomorrow's world. First, the traditional model assumes that the criteria in value-creation are independent and hierarchical in structure; however, criteria are often interdependent in real-world problems because 'Some statistics and economics assumptions are unrealistic in the real world'. The Decision Making Trial and Evaluation Laboratory (DEMATEL) technique is an effective tool to find the interrelationship matrix and building an influential network relation map (INRM) for solving relationship problems in the real world. Second, the relatively good solution from existing alternatives is replaced by aspiration levels to avoid "Choosing the best among inferior options/alternatives", i.e. "Picking the best apple among a barrel of rotten apples". Third, the emphasis in the field has shifted from ranking and selection when determining the most preferable approaches to performance improvement of existing methods based on INRM because "we need a systematic approach to problem-solving; instead of addressing the


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Talk: New Concepts and Trends of Hybrid MCDM Model for Tomorrow

- **New concepts and trends of hybrid MCDM model for Tomorrow**
- **How consider for solving the real world**
- **Basic concepts of ideas and thinking in trends**
- **New concepts and trends of hybrid MCDM model for Tomorrow: Some examples for the real cases**
  - Rough set theory (RST), Dominance-based rough set approach (DRSA) MCDM
  - MADM: DEMATEL, DANP (DEMATEL-based ANP), Integration (Additive: SAW, VIKOR, Grey Relation Analysis, PROMETHEE, ELECTRE; Non-additive: Fuzzy Integral)
  - MODM: Changeable Spaces Programming
- **Conclusions**

How consider for solving the real world problems

⇓

How consider for solving the real world problems

"Story (Objects)" in Real Case Problems
(Case Study in Experience) for Real World

+

Research Methods for Problems-Solving
(Which methods? New concepts and trends of hybrid MCDM model for Tomorrow)

⇓

Expressions in Results

(Writing Skills and Speech Skills in Logic)

Problems-Solving in a real world

How Do Logic Thinking and Reasoning?

Define and purpose

- Understand the addressed real world problems
- From conceptual ideas to symbolic notations based on logic reasoning in real world problems for achieving or toward **aspiration level**

Model

- Find and define relevant features/structures (variables/factors/aspects/dimensions/criteria/attributes by literature review, brain storming with Delphi, pre-testing)
- Propose an initial model for addressed on solving the real world problems (new concepts and trends of hybrid MCDM model (MRDM, MADM, and MODM) for solving real world problems)
- Apply suitable methods/techniques to form the real world model in real world case

Goal

- Description (e.g. linear, non-linear, logical, how interrelationship, dependence and feedback, etc.)
- Rule-based if-then data-mining (Forecast/prediction), evaluation (Ranking, selection, and improvement) for achieving or toward aspiration level and Plan/Design (Changeable spaces including decision space and objective space) planning/ programming in improvement for achieving or toward aspiration level
- Change traditional optimization solution based on resource constricts finding Pareto optimal solution (called optimization) into how improvement for achieving or toward aspiration level solution (or called the best solution) through innovation and creativity

Understanding

- Observation (experience)
- Intuition/feeling
- Knowledge/experts in practical experience
- Theory

Symbolic notation

- Conceptualize
  - Multiple dimensions/aspects
  - Multiple criteria/attributes/variables
  - Single or multiple objectives/goals
- Define data sets
  - Crisp data sets
  - Fuzzy sets
  - Rough sets,
  - Grey Hazy sets

- **Data** sets → **Information** Systems, $IS = (U, A, V, f)$

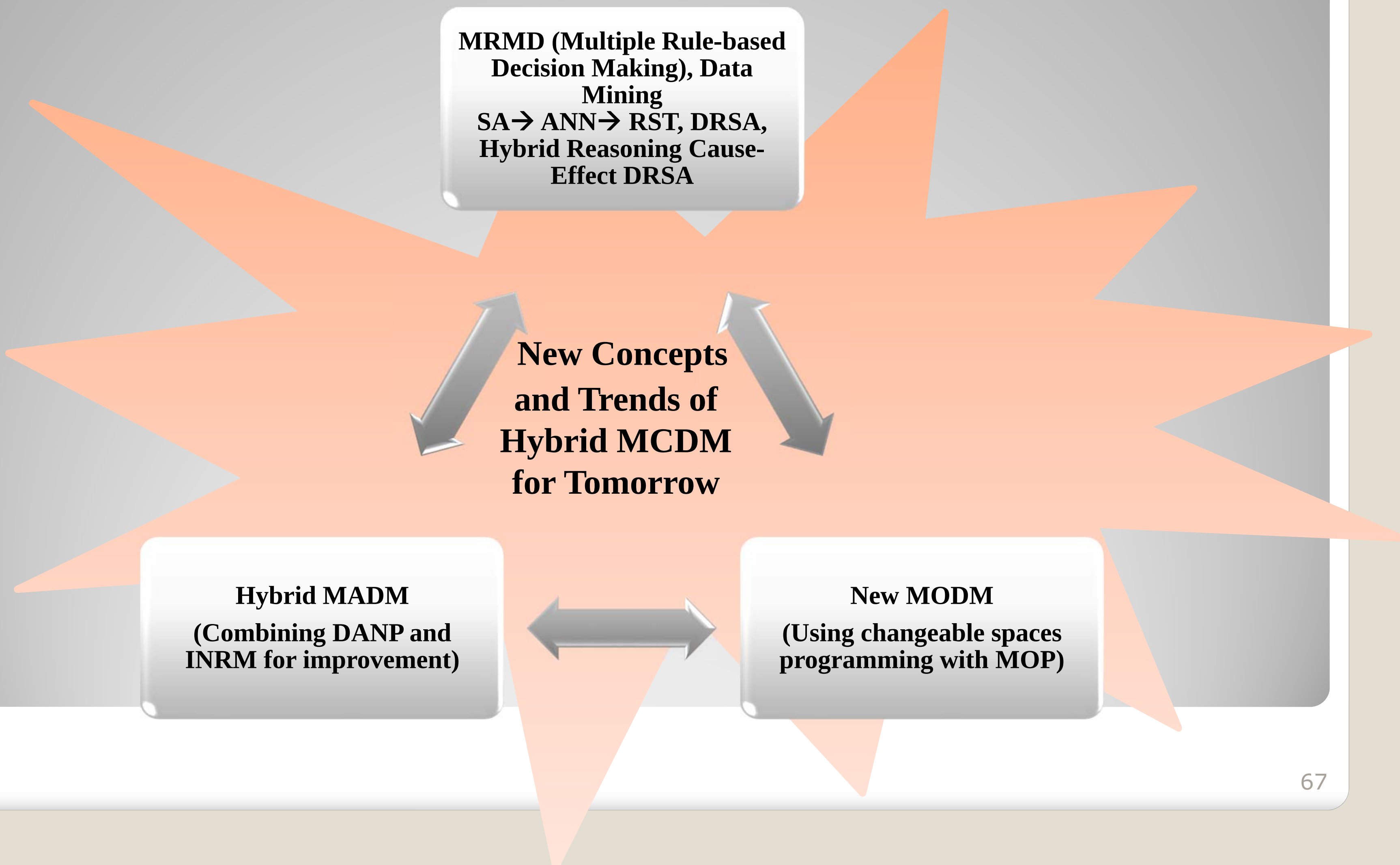
→ **Knowledge** Discovery → **Intelligence**/**Wisdom** (→ **enlightenment** for making the best decision based on new hybrid MADM and MODM)

Define a problem

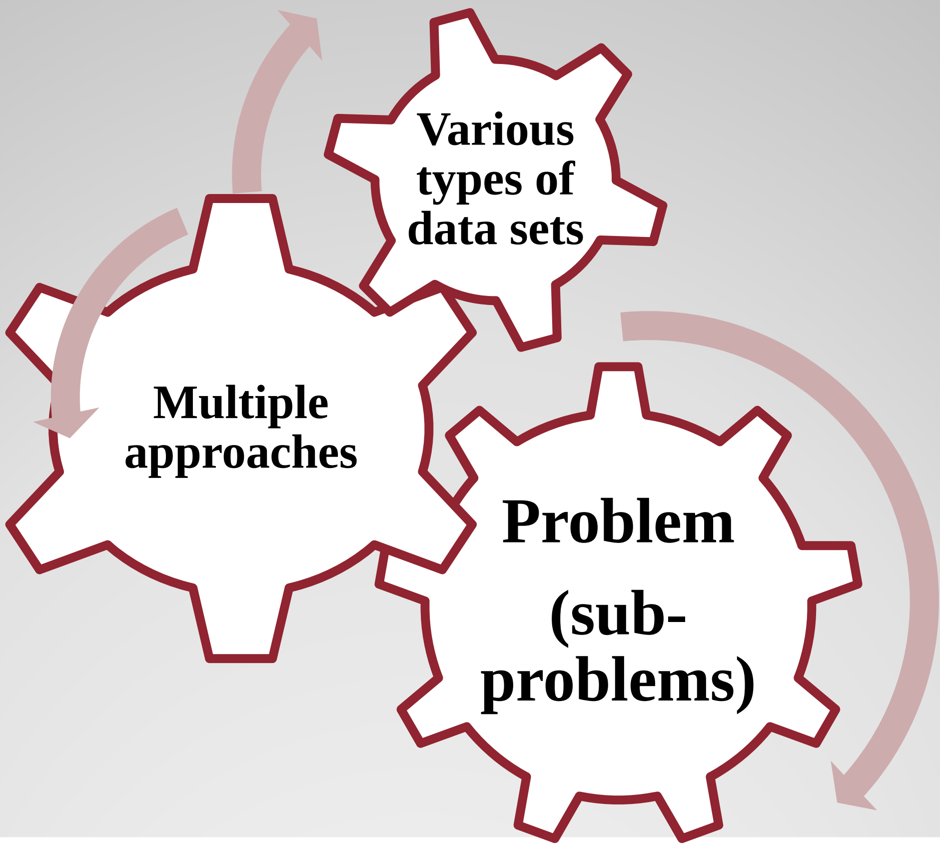
Multi-paths to reach the results

Statistical Analysis (SA) (Conventional approach)	Data sets (nominal, ordinal, scale, numerical)	Find/Shape the relations (factors, classification, regression, discriminant, MDS, etc.)
Evolutionary Computation (Evolutionary Algorithm, Soft Computing (Computation Intelligence))	Data sets (similar to SA, plus Fuzzy/Rough/Grey and Machining Computation)	Find/Shape the relations (classification, pattern, regression, discriminant, etc.)
New Hybrid MCDM (Hybrid MRDM/MADM/MODM approach)	Data sets (similar to SA, plus Fuzzy/Rough/Grey..)	Find/Shape the influence relationship (if-then cause-effect rule-based decision making for improvement, etc.)

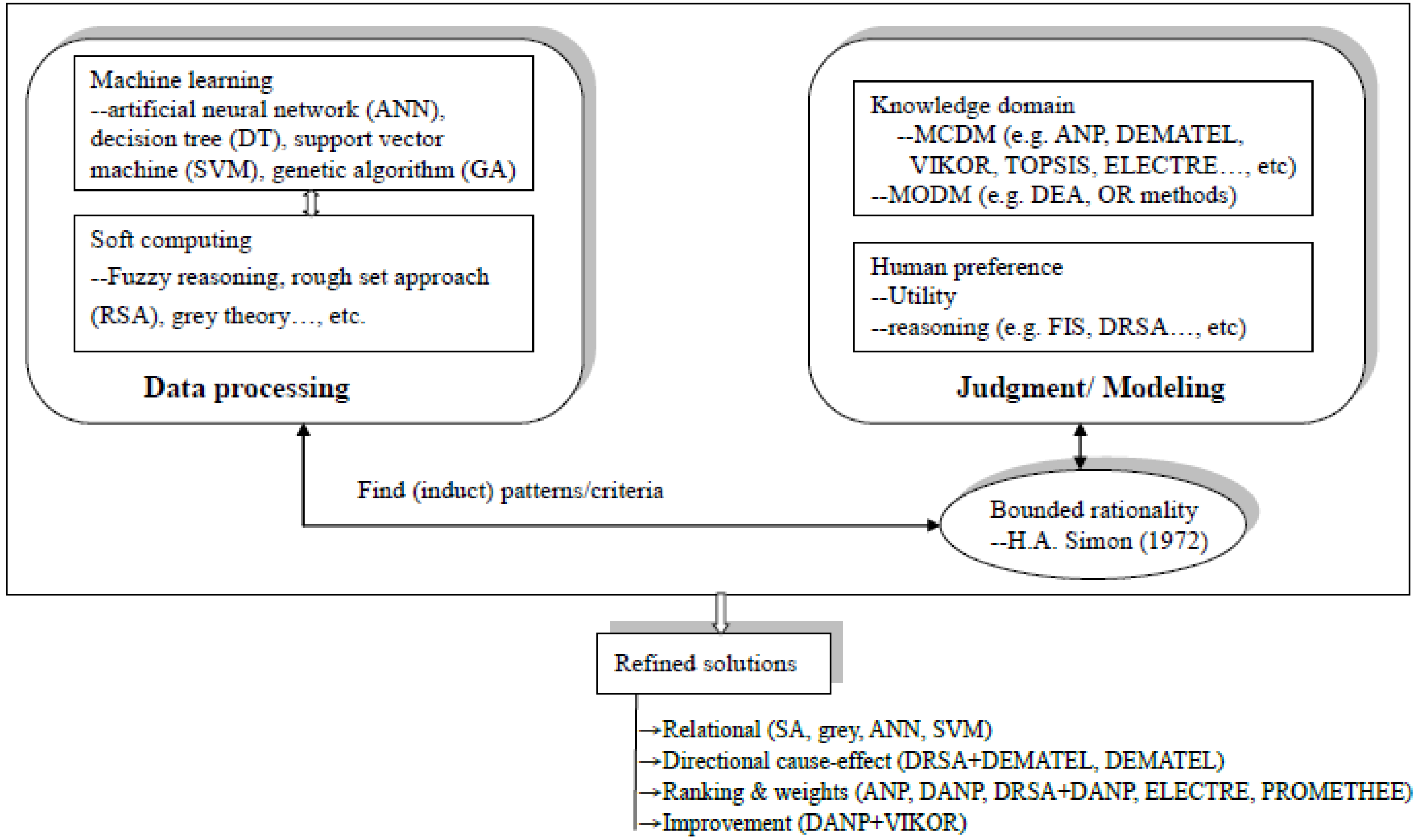
A new combined/hybrid MCDM approach for improving performance gaps



Infusion Information (Big Data by Complex Interrelationship)



Basic Concepts of Recent developments



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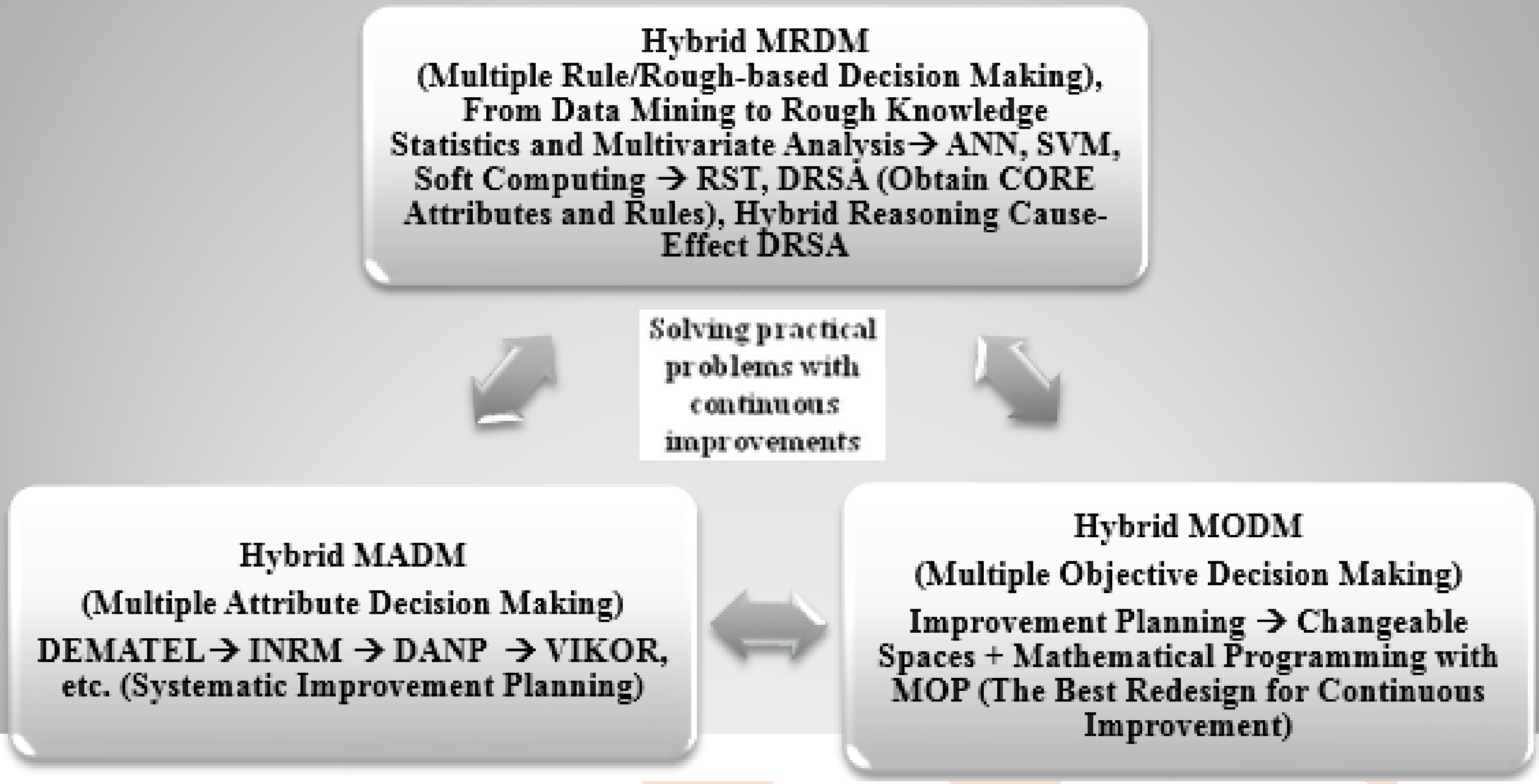
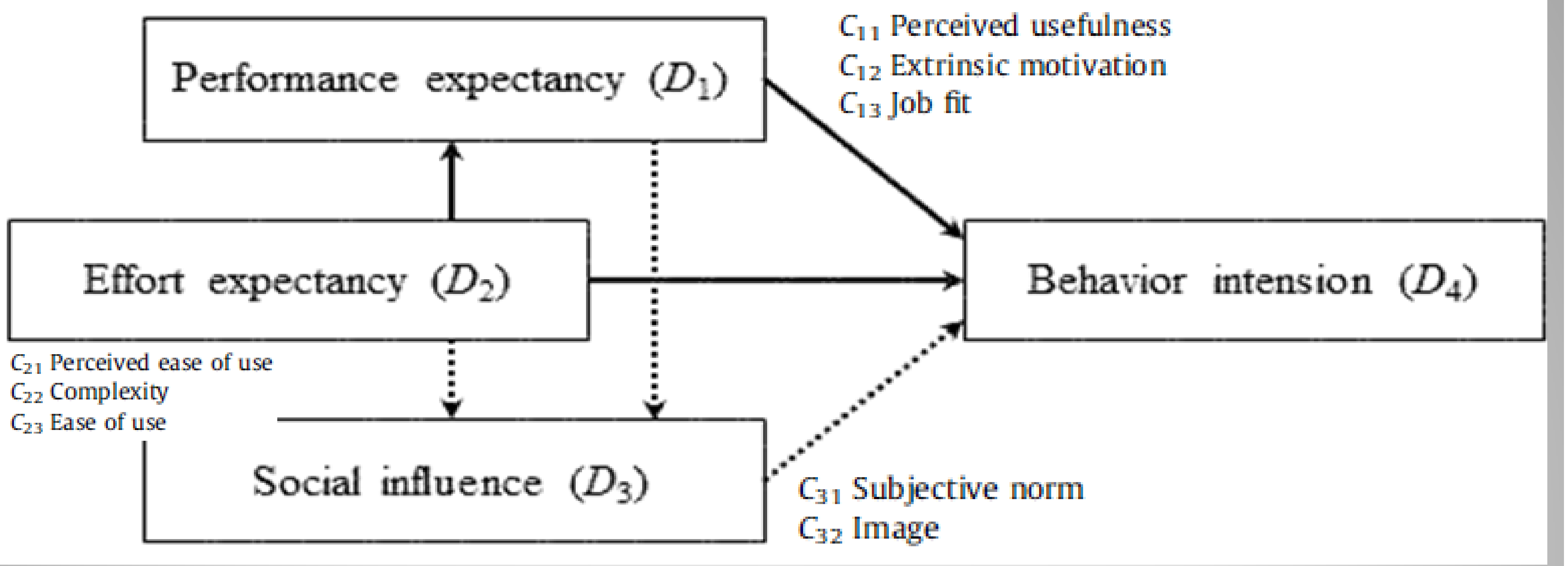
- Conclusions

Basic concepts of ideas and thinking in trends

A new concepts and trends of combined/hybrid MCDM approach for improving performance planning

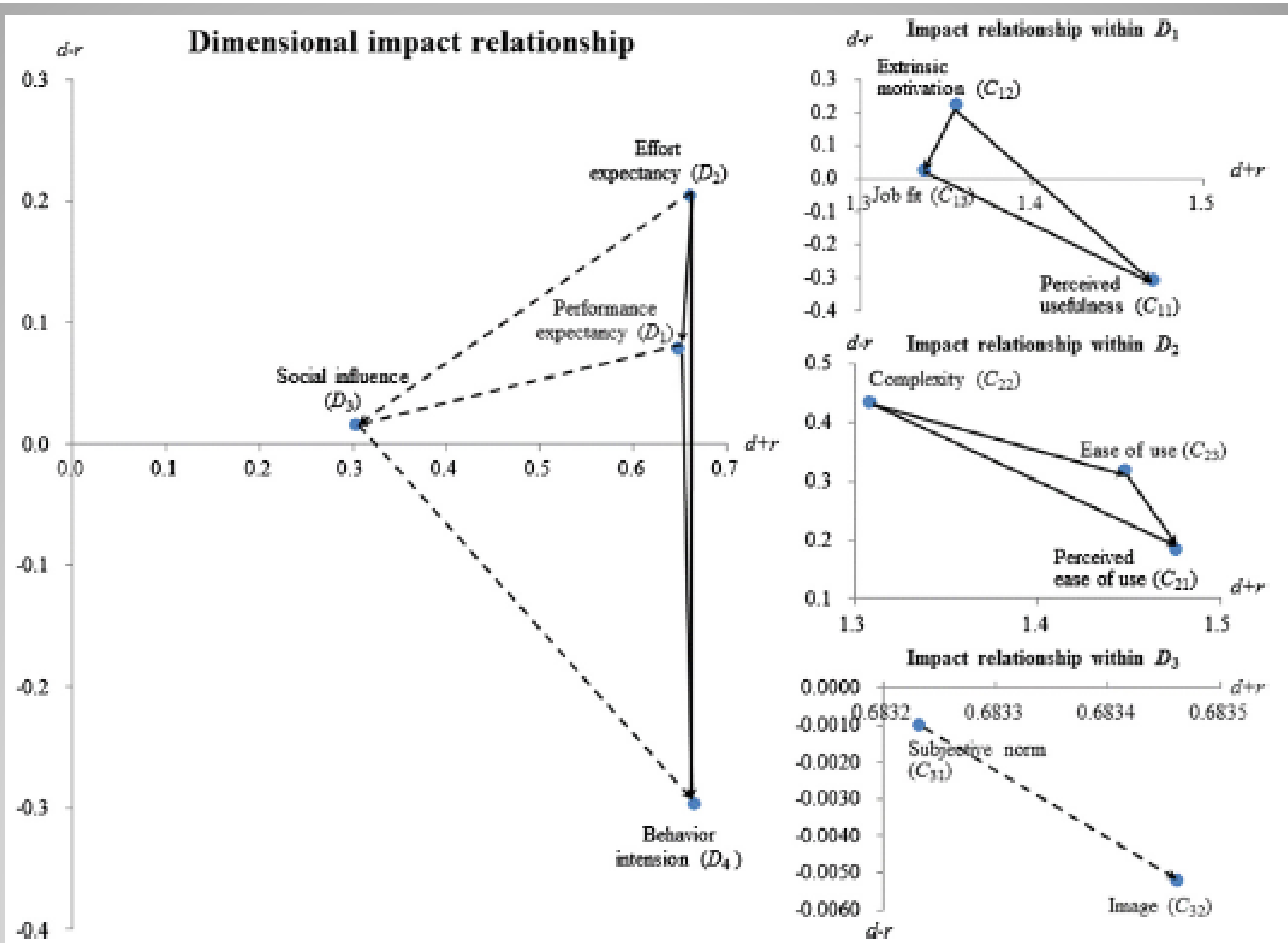


Basic concept of SEM (Structure Equation Modelling) combining DEMATEL technique



The causal relation map (SEM based on DEMATEL technique → DRSA → MCDM)

Jeng, D. J.F. and Tzeng, G.H. (2012). Social influence on the use of Clinical Decision Support Systems: Revisiting the Unified Theory of Acceptance and Use of Technology by the fuzzy DEMATEL technique, Computers & Industrial Engineering, 62(3), 819-828.



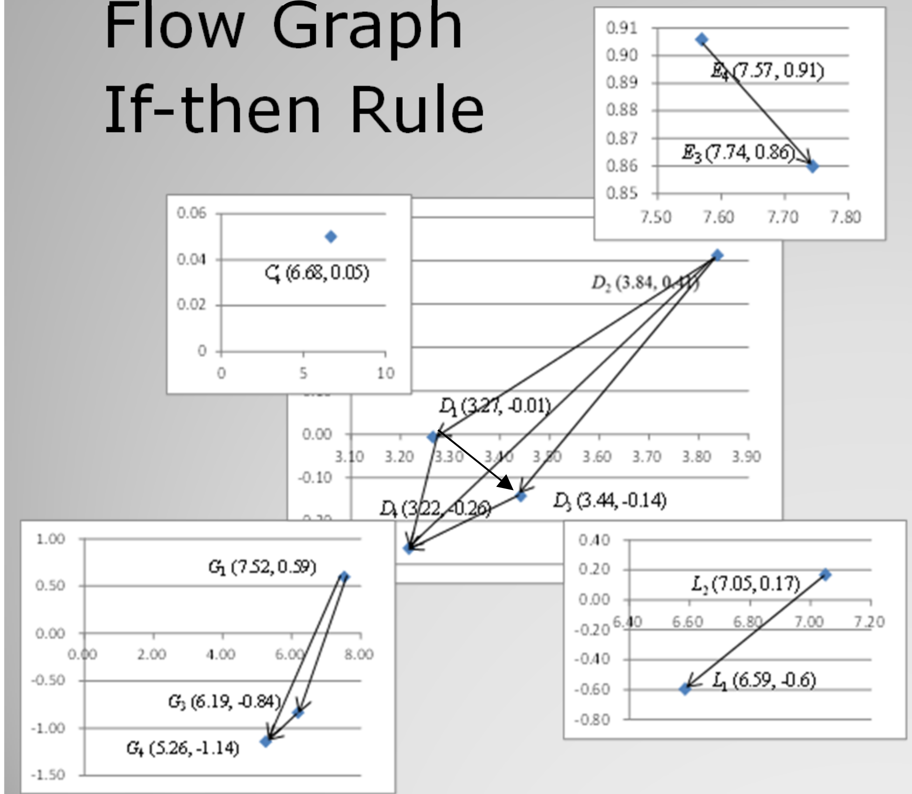
Influential network relation map (INRM) by DEMATEL technique

DRSA-based Neuro-Fuzzy Inference Systems for the Financial Performance Prediction of Commercial Bank

- DRSA → Cause-Effect

  Flow Graph

  If-then Rule



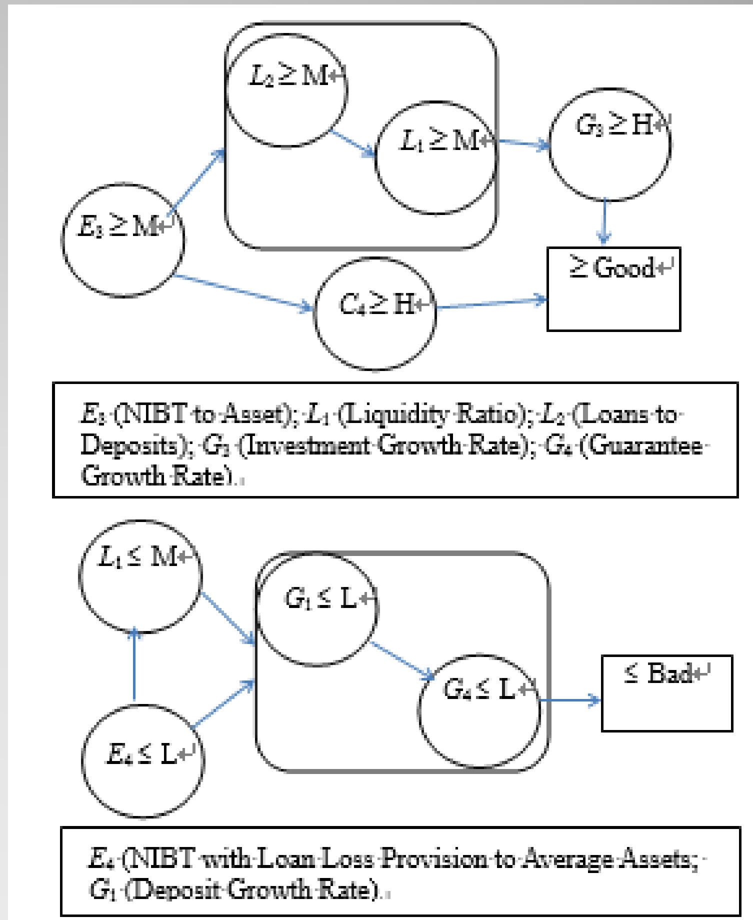
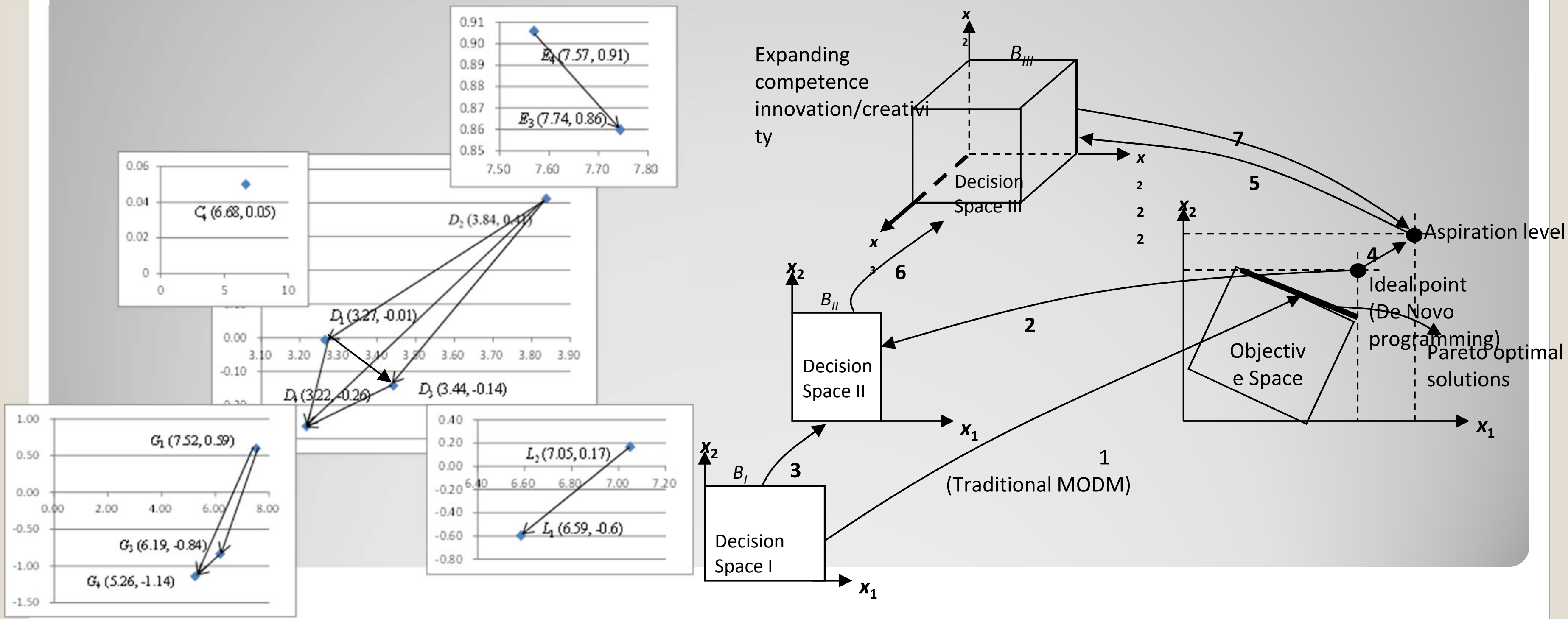
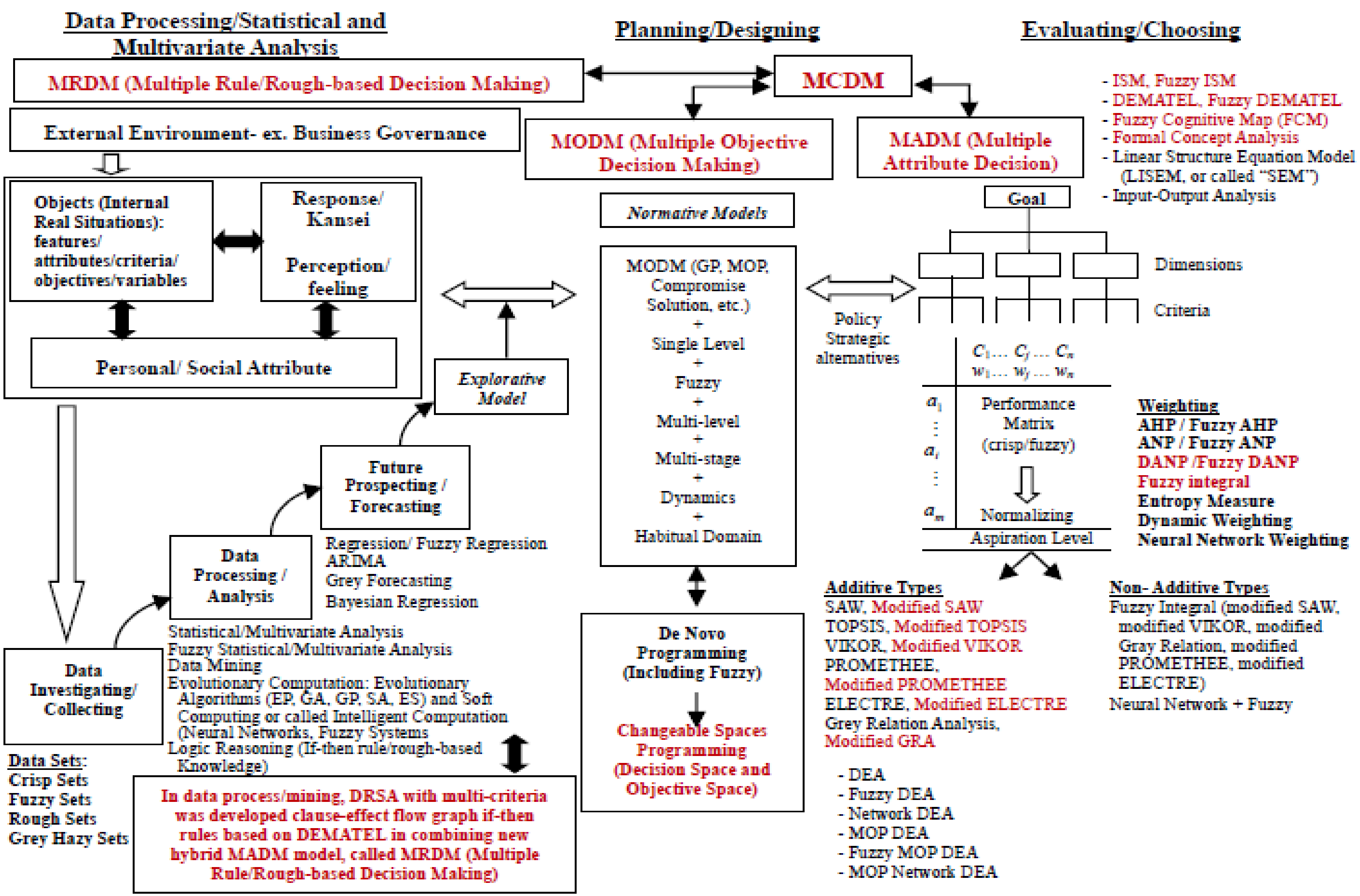


Figure 3. Directional Flow Graph

Kao-Yi Shen, and **Gwo-Hshiung Tzeng (2014).** DRSA-based Neuro-Fuzzy Inference Systems for the Financial Performance Prediction of Commercial Bank, *International*

How resource allocation to improve the gaps of performance values in each criterion for achieving aspiration level (MADM) through the basic concepts of changeable spaces (decision space and objective space) programming (MODM)

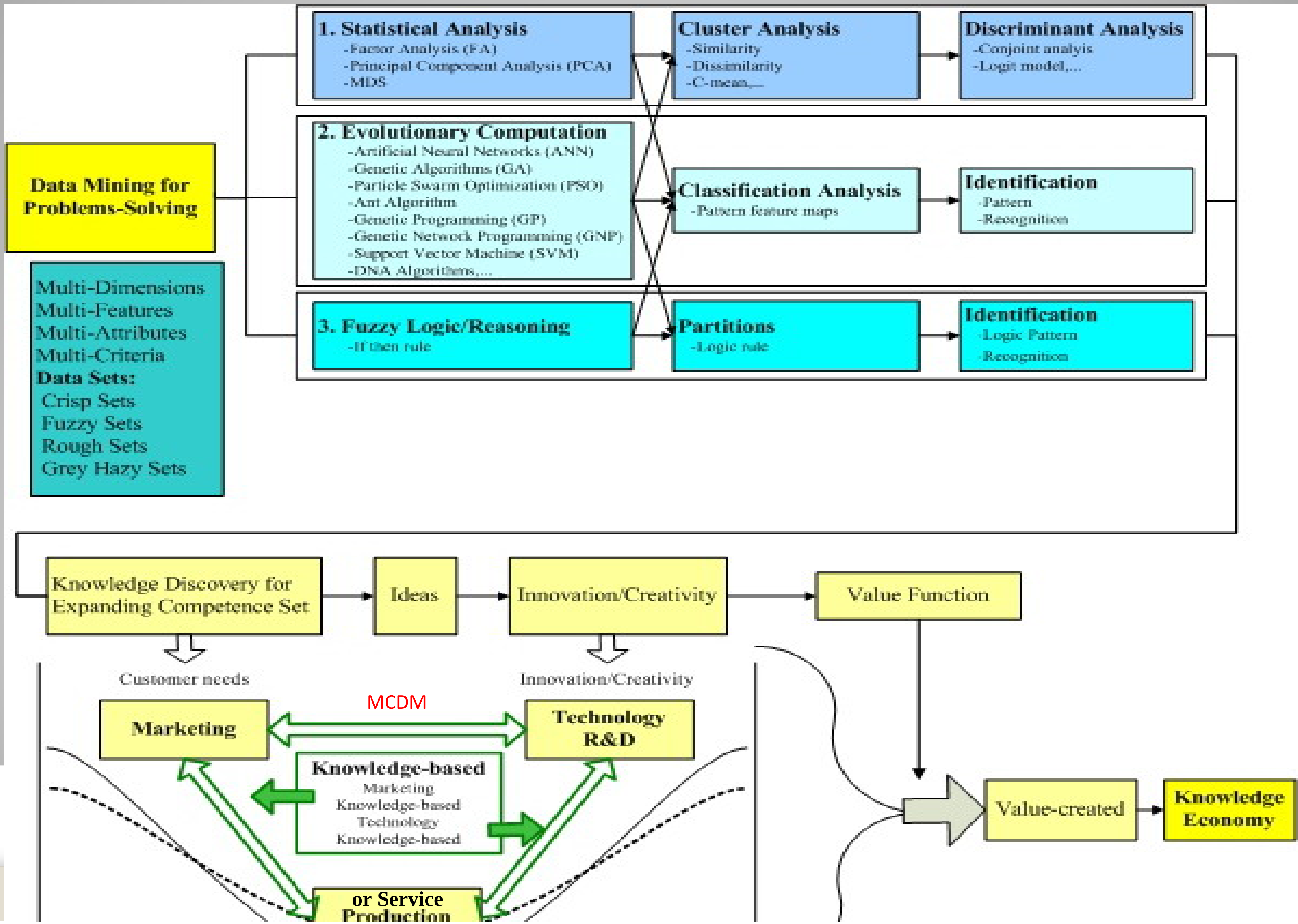




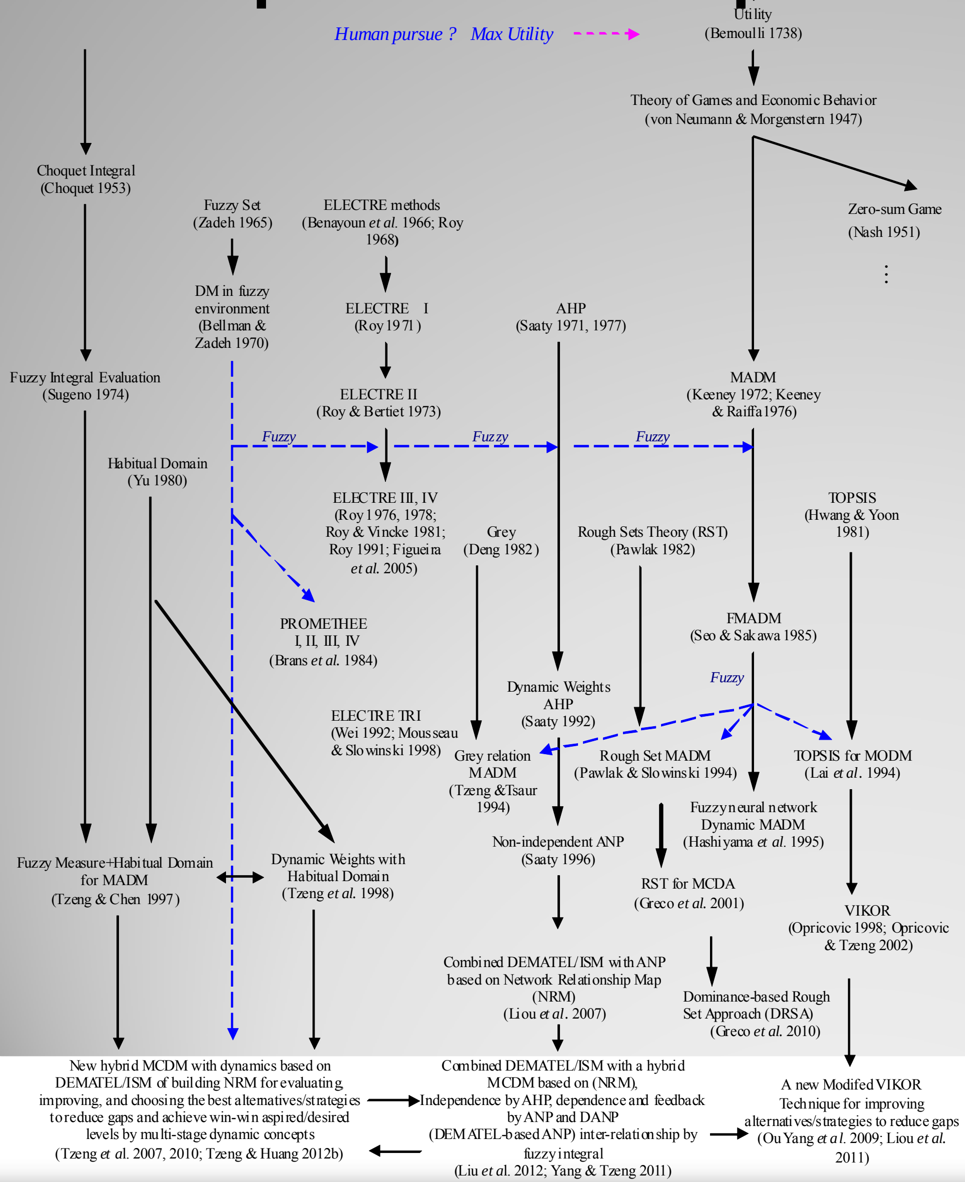
Basic Concepts of Overall Social Science "Research Methods" for Problem-Solving

(Tzeng and Huang, 2011, 2013; Liou and Tzeng, 2012; Peng and Tzeng, 2013)

Data Mining Concepts of Intelligent Computation in Knowledge Economy



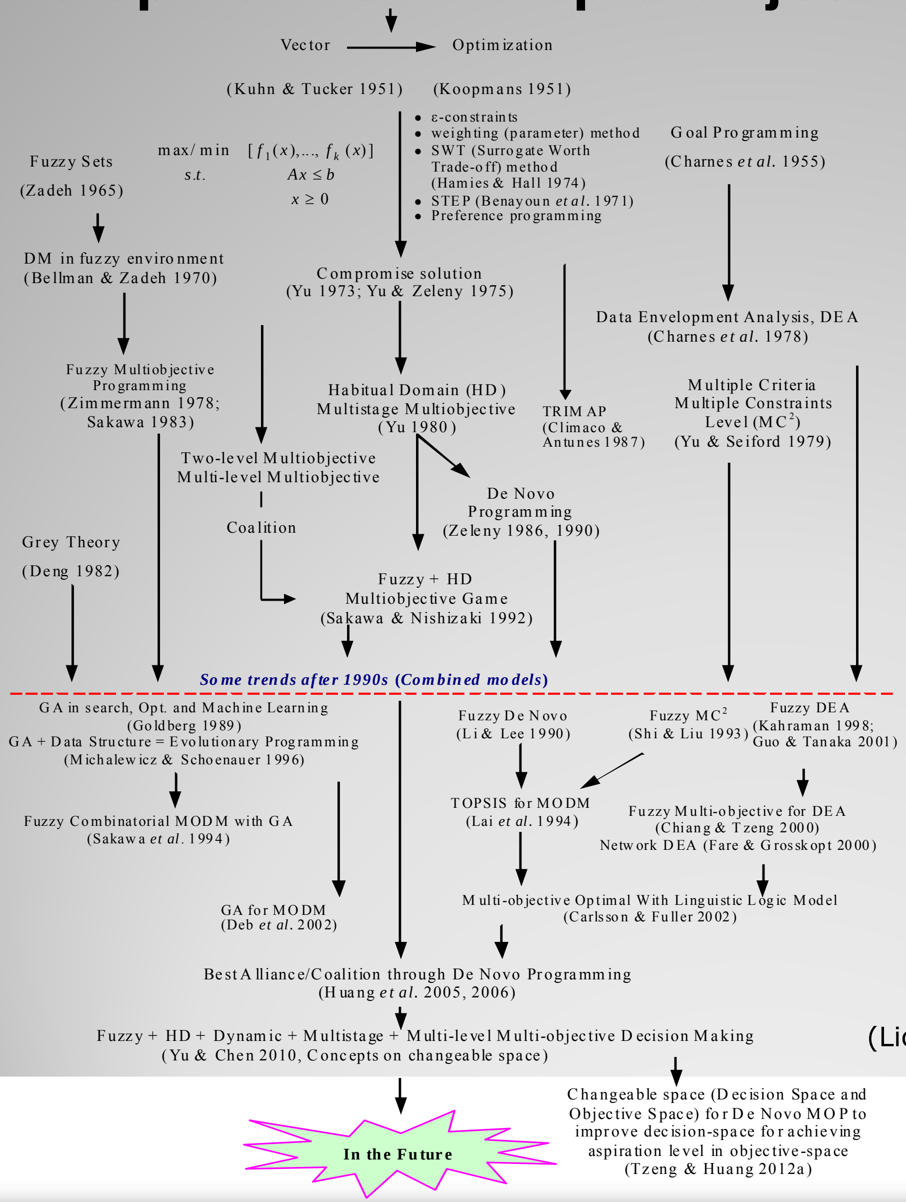
Development of Multiple Attribute Decision Making



(

(Liou and Tzeng, 2012; Huang and Tzeng, 2011, 2013)

Development of Multiple Objective Decision Making



(Liou and Tzeng, 2012; Huang and Tzeng, 2011, 2013)

Talk: New Concepts and Trends of Hybrid MCDM Model for Tomorrow

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  - MODM: Changeable Spaces Programming
- **Conclusions**

New concepts and trends of hybrid MCDM model for Tomorrow: Some examples for the real cases

- Rough sets theory (RST), Dominance-based rough set approach (DRSA) MCDM → Hybrid MRDM
- MADM: DEMATEL, DANP (DEMATEL-based ANP), Integration (Additive: SAW, VIKOR, Grey Relation Analysis, PROMETHEE, ELECTRE; Non-additive: Fuzzy Integral) → Hybrid Modified MADM
- MODM → New Hybrid MODM: Changeable Spaces Programming

New concepts and trends of hybrid MCDM model for Tomorrow: Some examples for the real cases

- Rough sets theory (RST), Dominance-based rough set approach (DRSA) MCDM → Hybrid MRDM
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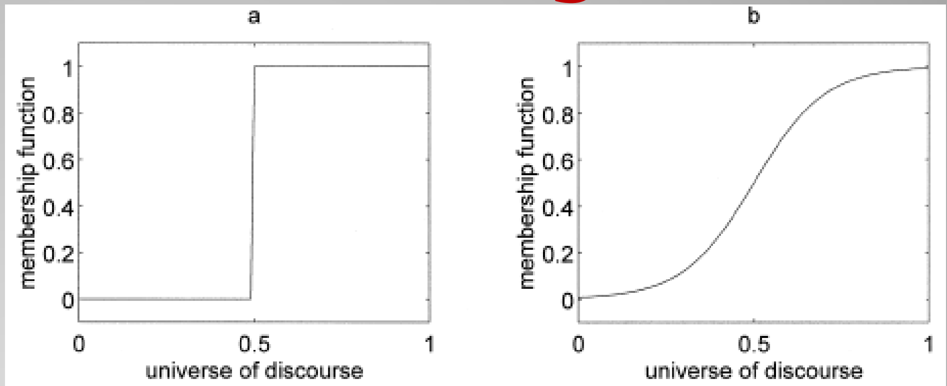
Basic concepts of the rough sets



Crisp set and Fuzzy set

Fuzzy set v.s. Rough set

- Fuzziness measures the degree to which an event occur and not whether it occurs
- Membership measures the degree of belong
- Fuzzy set is based on vagueness (no crisp in boundaries); Fuzzy measures based on ambiguity (many kind possibility don't know which); Rough set is based on indiscerniblity
- Rough set approach can be considered as a formal framework for discovering facts from imperfect data
- The results of the rough set approach are presented in the form of classification or decision rules derived from a set of examples

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Information Systems

An *information system*, *IS* or an approximation space., can be seen as a system,

$$IS = (U, A, V, f)$$

where *U* is the universe (a finite set of objects $\{x_1, x_2, \ldots, x_m\}$) and *A* is the set of attributes (or called features, criteria, elements, factors, variables, and so on). Each attribute $a \in A$ (attribute a belonging to the considered set of attributes *A*) defines an information function $f_a: U \to V_a$, where V_a is the set of values of a, called the domain of attribute a.

Example 1

Consider a data set containing the results of three measurements performed for 10 objects. The results can be organized in a matrix 10x3.

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

2	1	3
3	2	1
2	1	3
2	2	3
1	1	4
1	1	2
3	2	1
1	1	4
2	1	3
3	2	1

Using the terminology of the rough sets theory, this data set can be considered as an information system $IS = (U, A, V, f)$, where universe *U* and attributes *A* correspond to the set of objects and to the set of variables, respectively:

$$U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$$

$$A = \{a_1, a_2, a_3\}$$

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

The domains of the particular attributes are:

$V_1 = \{1,2,3\}$, $V_2 = \{1,2,\}$, $\{1,2,3,4\}$,

i.e., the domain of each attribute is the set of values of this attribute. The information function f_a for this system is presented in Table 1.

Table 1 $f_a : U \rightarrow V_a$ in $IS = (U, A, V, f)$ and $A = \{a_1, a_2, a_3\}$

U	a_1	a_2	a_3
x_1	2	1	3
x_2	3	2	1
x_3	2	1	3
x_4	2	2	3
x_5	1	1	4
x_6	1	1	2
x_7	3	2	1
x_8	1	1	4
x_9	2	1	3
x_{10}	3	2	1

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

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x_6	1	1	2
x_7	3	2	1
x_8	1	1	4
x_9	2	1	3
x_{10}	3	2	1

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Indiscernibility relation (1/2)

For every set of attributes $B \subset A$, an *indiscernibility relation* $Ind(B)$ is defined in the following way: two objects, x_i and x_j, are indiscernible by the set of attributes B in A, if $b(x_i)=b(x_j)$ for every $b \in B$. The equivalence class of $Ind(B)$ is called *elementary set* in *B* because it represents the smallest discernible groups of objects. For any element x_i of *U*, the equivalence class of x_i in relation $Ind(B)$ is represented as $[x_i]_{Ind(B)}$. The construction of elementary sets is the first step in classification with rough sets.

Example 2

As one can easily notice, there are some identical objects in our data set. For instance, objects x_1 and x_3 cannot be distinguished based on the available data.

Let us group all objects based on the three variables considered. The results are presented in **Table 2**.

Table 2

U/A	a_1	a_2	a_3
$\{x_1, x_3, x_9\}$	2	1	3
$\{x_2, x_7, x_{10}\}$	3	2	1
$\{x_4\}$	2	2	3
$\{x_5, x_8\}$	1	1	4
$\{x_6\}$	1	1	2

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

<u>Indiscernibility relation</u> (2/2)

Each row in this table describes one elementary set, whereas the whole table describes the IS studied. The notation *U*/*A* means that we are considering elementary sets of the universe *U* in the space *A*.

It can happen that we are interested in the two attributes only, for instance in a_1 and a_2. Then the indiscernibility relation is limited to the subset $B = \{a_1, a_2\}$ and the resulting elementary sets are given in Table 3.

Table 3

U/B	a_1	a_2
$\{x_1, x_3, x_9\}$	2	1
$\{x_2, x_7, x_{10}\}$	3	2
$\{x_4\}$	2	2
$\{x_5, x_6, x_8\}$	1	1

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Lower and upper approximations (1/5)

The rough sets approach to data analysis hinges on two basic concepts, namely the *lower* and the *upper approximations* of a set **Fig. 2.**, referring to:

-the elements that doubtlessly belong to the set, and

-the elements that possibly belong to the set.

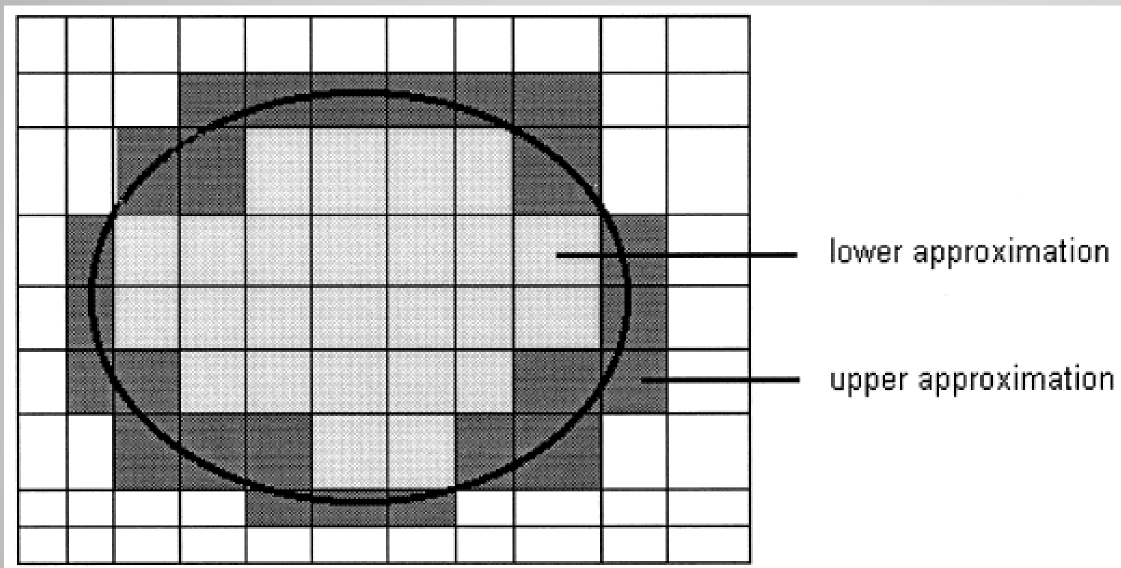

Fig. 2. Schematic demonstration of the upper and lower approximation of set *X*.

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

<u>Lower and upper approximations</u> (2 / 5)

Let *X* denote the subset of elements of the universe *U* $(X \subseteq U)$. The lower approximation of *X* in *B* $(B \subseteq A)$, denoted as $\underline{BX}$, is defined as the union of all these elementary sets which are contained in *X*. More formally:

$\underline{BX} = \{x_i \in U | [x_i]_{Ind(B)} \subset X\}$.

The above statement is to be read as: the lower approximation of the set *X* is a set of objects x_i, which belong to the elementary sets contained in *X* (in the space *B*).

The upper approximation of the set *X*, denoted as BX, is the union of these elementary sets, which have a non-empty intersection with *X*:

$\overline{BX} = \{x_i \in U | [x_i]_{Ind(B)} \cap X \neq \emptyset\}$.

For any object x_i of the lower approximation of $X\ (i.e., x_i \in \underline{BX})$, it is certain that it belongs to *X*. For any object x_i of the upper approximation of $X\ (i.e., x_i \in \overline{BX})$, we can only say that x_i may belong to *X*. The difference:

$BNX = \overline{BX} - \underline{BX}$

is called a boundary of *X* in *U*.

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Lower and upper approximations (3/5)

If the lower and upper approximation are identical $(i.e., \overline{BX} = \underline{BX})$, then set X is definable, otherwise, set X is undefinable in U. There are four types of undefinable sets in U:

1. if $\underline{BX} \neq \emptyset$ and $\overline{BX} \neq U$, X is called roughly definable in U;
2. if $\underline{BX} \neq \emptyset$ and $\overline{BX} = U$, X is called externally undefinable in U;
3. if $\underline{BX} = \emptyset$ and $\overline{BX} \neq U$, X is called internally undefinable in U;
4. if $\underline{BX} = \emptyset$ and $\overline{BX} = U$, X is called totally undefinable in U,

where $\emptyset$ denotes an empty set.

Additionally, the following notation can be introduced: $POS_B(X) = \underline{BX}$, called the B-positive region B of X, is the set of these objects, which can, with certainty, be classified in the set X, $NEG_B(X) = U - \underline{BX}$, called the B-negative region of X, is the set of objects, which without ambiguity, can be classified as belonging to the complement of X or as not belonging to X ., $BN_B(X)$, called the B-borderline B region of X, is an undecidable area of the universe, i.e., none of the objects belonging to the boundary can, with certainty, be classified into X or $-X$, as far as the attributes B are considered.

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Lower and upper approximations (4 / 5)

Example 3

Let us assume that we are interested in the subset *X* of five objects. $(X = \{x_1, x_3, x_4, x_5, x_9\})$. Can we distinguish this set from the whole data set in the space of three attributes $(B = \{a_1, a_2, a_3\})$? Based on the results presented in Table 2, one can calculate the lower and upper approximations of this set in the following Way.

The elementary sets presented in Table 2, which are also contained in *X*, are: $\{x_1, x_3, x_9\}, \{x_4\}$.

It means that the lower approximation is given by the following set of objects: $\underline{BX} = \{x_1, x_3, x_4, x_9\}$.

To calculate the upper approximation of the subset *X*, one has to find in Table 2 all elementary sets which have at least 1 element in common with the subset *X*. These are: $\{x_1, x_3, x_9\}$, $\{x_4\}$, $\{x_5, x_8\}$, so that the upper approximation is:

$\overline{BX} = \{x_1, x_3, x_4, x_5, x_8, x_9\}$.

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Lower and upper approximations (5/5)

The boundary of *X* in *U*, defined as the difference between the upper and lower approximations, contains elements which are in the upper but not in the lower approximation:

$$BNX = \{x_1, x_3, x_4, x_5, x_8, x_9\} - \{x_1, x_3, x_4, x_9\}$$
$$= \{x_5, x_8\}$$

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Accuracy of approximation (1/1)

An accuracy measure of the set *X* in *B*:*A* is defined as:

$$\mu_B(X) = card(\underline{B}X) / card(\overline{B}X)$$

The cardinality of a set is the number of objects contained in the lower upper. approximation of the set *X*. As one can notice, $0 \leq \mu_B(X) \leq 1$. If *X* is *B* definable in *U* then $\mu_B(X) = 1$, if *X* is undefinable *B* in *U* then $\mu_B(X) < 1$.

Example 4

The number of objects contained in the lower approximation of example 3 equals 4. The cardinality of the upper approximation equals 6. The accuracy of set *X* therefore is: $\mu_B(X) = 4/6$.

This means that the considered set *X* is roughly definable in *U*; it can be defined by its lower and upper approximations in *U*.

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Independence of attributes (1/2)

In order to check, whether the set of attributes is independent or not, one checks for every attribute whether its removal increases the number of elementary sets in the IS or not.

If $Ind(A)\_ = Ind(A - a_i)$, then the attribute a_i is called superfluous. Otherwise, the attribute a_i is dispensable in *A*.

Example 5

Consider Table 1. If the three attributes $(a_1, a_2\ and\ a_3)$ are taken into account, five elementary sets can be constructed (see Table 2). Table 4 gives the number of elementary sets after leaving out one of the attributes. For instance, if only a_2 and a_3 are used, five elementary sets are distinguished, if a_1 and a_3 are used, the number of elementary sets is 4.

If we remove attribute a_2 or a_3, the number of elementary sets becomes smaller, but by removing attribute a_1, we do not change the elementary sets. Attribute a_1 is superfluous, whereas attributes a_2 and a_3 are indispensable.

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

<u>Independence of attributes</u> (2/2)

Table 4

	Removed attribute			
	None	a_1	a_2	a_3
Number of elementary sets	5	5	4	4

The set of attributes is dependent because by removing attribute a_1, we obtain the information system identical with that presented in Table 2.

Elimination of superfluous attributes simplifies the information set and has diagnostic value. It should be noted here that later, we will define so-called *D*-superfluous attributes. This definition is used for classification purpose.

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

<u>Core and reduct of attributes</u> (1/4)

If the set of attributes is dependent, one can be interested in finding all possible minimal subsets of attributes, which lead to the same number of elementary sets as the whole set of attributes *reducts*. And in finding the set of all indispensable attributes (*core*).

The concepts of core and reduct are two fundamental concepts of the rough sets theory. The reduct is the essential part of an IS, which can discern all objects discernible by the original IS. The core is the common part of all reducts. To compute reducts and core, the *discernibility matrix* is used. **<u>Discernibility matrix</u>** has the dimension nXn, where n denotes the number of elementary sets and its elements are defined as the set of all attributes which discern elementary sets $[x]_i$ and $[x]_j$.

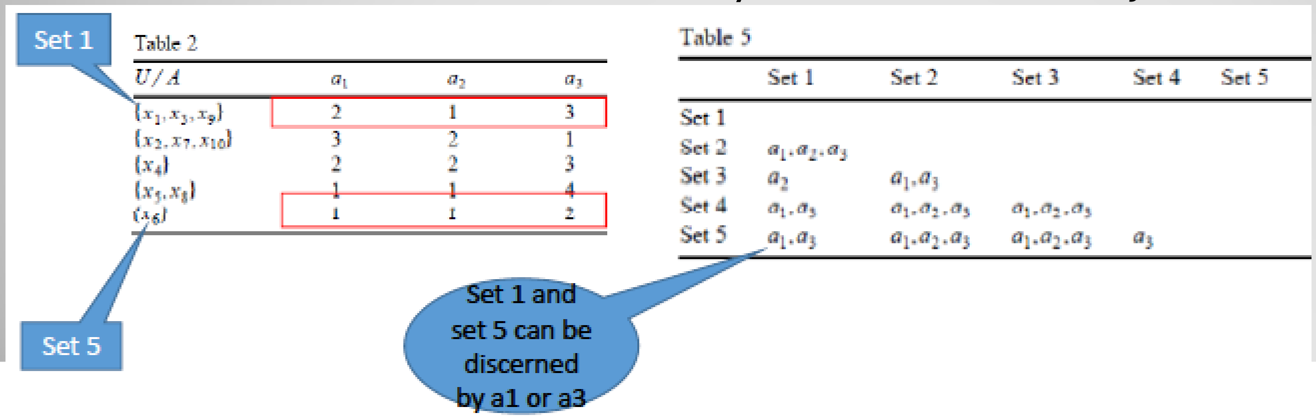

Table 2

U/A	a_1	a_2	a_3
$\{x_1, x_3, x_9\}$	2	1	3
$\{x_2, x_7, x_{10}\}$	3	2	1
$\{x_4\}$	2	2	3
$\{x_5, x_8\}$	1	1	4
$\{x_6\}$	1	1	2

Table 5

	Set 1	Set 2	Set 3	Set 4	Set 5
Set 1					
Set 2	a_1, a_2, a_3				
Set 3	a_2	a_1, a_3			
Set 4	a_1, a_3	a_1, a_2, a_3	a_1, a_2, a_3		
Set 5	a_1, a_3	a_1, a_2, a_3	a_1, a_2, a_3	a_3	

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

<u>Core and reduct of attributes</u> (2/4)

Table 2

U/A	a_1	a_2	a_3
$\{x_1, x_3, x_9\}$	2	1	3
$\{x_2, x_7, x_{10}\}$	3	2	1
$\{x_4\}$	2	2	3
$\{x_5, x_8\}$	1	1	4
$\{x_6\}$	1	1	2

<u>Example 6</u>

The discernibility matrix, *D*, for the five elementary sets presented in Table 2, is constructed in the following way. To calculate element d_{ij}, one ought to find the set of attributes which discern the elementary sets *i* and *j*. The set of attributes which discern the elementary sets 1 and 2 contains attributes a_1, a_2, and a_3, i.e., $d_{21} = d_{12} = \{a_1, a_{2,} a_3\}$. The element $d_{31} = d_{13} = \{a_2\}$, i.e., the attribute a_2 only discerns the elementary sets 3 and 1. As the discernibility matrix is symmetrical $(d_{ij} = d_{ji})$, it is enough to consider its lower diagonal part only. Of course, each elementary set differs from the rest of elementary sets, due to at least one attribute, so that there are no empty cells in the discernibility matrix. The discernibility matrix can be used to find the minimal subsets. of attributes reducts, which leads to the same partition of the data as the whole set of attributes *A*. To do this, one has to construct the so-called discernibility function $f(A)$. This is a Boolean function, constructed in the following way:

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Basic concepts of the rough sets

Core and reduct of attributes (3 / 4)

Table 5

	Set 1	Set 2	Set 3	Set 4	Set 5
Set 1					
Set 2	a_1, a_2, a_3				
Set 3	a_2	a_1, a_3			
Set 4	a_1, a_3	a_1, a_2, a_3	a_1, a_2, a_3		
Set 5	a_1, a_3	a_1, a_2, a_3	a_1, a_2, a_3	a_3	

Example 6 (continuously)

To each attribute from the set of attributes, which dis-cern two elementary sets, (e.g., $\{a_1, a_2, a_3\}$), we assign a Boolean variable 'a', and the resulting Boolean function attains the form $(a_1 + a_2 + a_3)$ (or it can be presented as $(a_1 \vee a_2 \vee a_3)$). If the set of attributes is empty, we assign to it the Boolean constant 1. For the discernibility matrix presented in Table 5, the discernibility function has the following form:

$$f(A) = (a_1 + a_2 + a_3)a_2(a_1 + a_3)(a_1 + a_3) \times (a_1 + a_3)(a_1 + a_2 + a_3)(a_1 + a_2 + a_3) \times (a_1 + a_2 + a_3)(a_1 + a_2 + a_3)a_3$$

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Core and reduct of attributes (4/4)

Table 5

	Set 1	Set 2	Set 3	Set 4	Set 5
Set 1					
Set 2	a_1, a_2, a_3				
Set 3	a_2	a_1, a_3			
Set 4	a_1, a_3	a_1, a_2, a_3	a_1, a_2, a_3		
Set 5	a_1, a_3	a_1, a_2, a_3	a_1, a_2, a_3	a_3	

Example 6 (continuously)

To calculate the final form of f A., the absorption law is applied. According to the absorption law, if the elementary set 1 differs from the elementary set 2 due to the attributes a_1, a_2 and a_3, and from the elementary set 3 due to the attribute a_2, it is enough to take into the account the attribute a_2 only, which discerns this set from both set 2 and set 3, i.e:

$$(a_1 + a_2 + a_3)a_2 = a_2$$

Let us look at another example. Suppose that to discern the elementary set 1 from the sets 2, 3, 4 and 5, one has to take into account the following sets of attributes (see the first column in Table 5):

$$\{a_2\}, \{a_1 + a_2 + a_3\}, \{a_1, a_3\} \text{ and } \{a_1, a_3\}$$

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Basic concepts of the rough sets

Core and reduct of attributes value

$f_1(A) = (a_2 + a_3) a_2 a_3 a_3 = a_2 a_3$

$f_2(A) = (a_2 + a_3) a_3 (a_2 + a_3)(a_2 + a_3) = a_3$

$f_3(A) = a_2 a_3 (a_2 + a_3)(a_2 + a_3) = a_2 a_3$

$f_4(A) = a_3 (a_2 + a_3)(a_2 + a_3) a_3 = a_3$

$f_5(A) = a_3 (a_2 + a_3)(a_2 + a_3)(a_2 + a_3) a_3 = a_3$

Consider $\{a_2, a_3\}$

The discernibility matrix as followed

Table 6

U/R	a_2	a_3
$\{x_1, x_3, x_9\}$	1	3
$\{x_2, x_7, x_{10}\}$	2	1
$\{x_4\}$	2	3
$\{x_5, x_8\}$	1	4
$\{x_6\}$	1	2

Table 7

	Set 1 (F1(A))	Set 2 (F2(A))	Set 3 (F3(A))	Set 4 (F4(A))	Set 5 (F5(A))
Set 1		a_2, a_3	a_2	a_3	a_3
Set 2	a_2, a_3		a_3	a_2, a_3	a_2, a_3
Set 3	a_2	a_3		a_2, a_3	a_2, a_3
Set 4	a_3	a_2, a_3	a_2, a_3		a_3
Set 5	a_3	a_2, a_3	a_2, a_3	a_3	

Table 8

U/R	a_2	a_3
$\{x_1, x_3, x_9\}$	1	3
$\{x_2, x_7, x_{10}\}$	*	1
$\{x_4\}$	2	3
$\{x_5, x_8\}$	*	4
$\{x_6\}$	*	2

*Denotes 'do not care'.

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Classification

- $F = \{X_1, X_2, \ldots, X_n\}$, $X_i \subset U$, and $X_i \cap X_j = \varnothing$

F is called classification of U, and X_i are called classes

- The lower and upper approximations of F in $B \subseteq A$ are defined as

$\underline{B}(F) = \{\underline{B}(X_1), \underline{B}(X_2), \ldots \underline{B}(X_n)\}$

$B(F) = \{B(X_1), B(X_2), \ldots B(X_n)\}$

- Quality of classification is defined as

$\eta_B F = (\cup \text{card } \underline{B}(X_i))/\text{card} U$

- Accuracy of classification F in B

$\beta_B F = \cup \text{ card } \underline{B}(X_i)/ \cup \text{ card B}(X_i)$

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Basic concepts of the rough sets

Decision table

- Class 1 = {x_1, x_3, x_9}, class 2 = {x_2, x_4, x_7, x_{10}}, class 3 = {x_5, x_6, x_8}

Condition attribute

Decision attribute

U	a_1	a_2	a_3	d
x_1	2	1	3	1
x_2	3	2	1	2
x_3	2	1	3	1
x_4	2	2	3	2
x_5	1	1	4	3
x_6	1	1	2	3
x_7	3	2	1	2
x_8	1	1	4	3
x_9	2	1	3	1
x_{10}	3	2	1	2

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Decision table

D-superfluous attribute

Attribute a_1, belonging to the condition set of at tributes B $(where\, B \subseteq A)$, is D-superfluous if it exerts no influence on the lower approximation of D, i.e., if

$$POS_B(D) = POS_{(B-a_i)}(D)$$

Otherwise, attribute a_i is *D*-indispensable in *A*.

Main steps of decision table analysis

- Construction of elementary sets in *D*-space
- Calculation of upper and lower approximations of the elementary sets in *D*
- Finding *D*-core and *D*-reduct of *A* attributes
- Finding *D*-core and *D*-reducts of A attribute values.

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

D-core and D-reduct (1/2)

Let us start with Table 12. In the *D*-space, one can find the following elementary sets:

set 1: $\{x_1, x_3, x_9\}$,
set 2: $\{x_2, x_4, x_7, x_{10}\}$,
set 3: $\{x_5, x_6, x_8\}$.

Table 2

U/A	a_1	a_2	a_3
$\{x_1, x_3, x_9\}$	2	1	3
$\{x_2, x_7, x_{10}\}$	3	2	1
$\{x_4\}$	2	2	3
$\{x_5, x_8\}$	1	1	4
$\{x_6\}$	1	1	2

Table 12

U	a_1	a_2	a_3	d_1	d_2
x_1	2	1	3	2	3
x_2	3	2	1	3	1
x_3	2	1	3	2	3
x_4	2	2	3	3	1
x_5	1	1	4	1	3
x_6	1	1	2	1	3
x_7	3	2	1	3	1
x_8	1	1	4	1	3
x_9	2	1	3	2	3
x_{10}	3	2	1	3	1

Table 14

Class number	Number of objects	Lower approximation	Upper approximation	Accuracy
1	3	3	3	1.0
2	4	4	4	1.0
3	3	3	3	1.0

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

D-core and D-reduct (2/2)

$$
\begin{aligned}
f_A(D) = {} & (a_1+a_2+a_3)a_2(a_1+a_3)(a_1+a_3) \\
& \times(a_1+a_2+a_3)(a_1+a_3)(a_1+a_2+a_3) \\
& \times(a_1+a_2+a_3)(a_1+a_3+a_3) \\
& \times(a_1+a_2+a_3)(a_1+a_2+a_3) \\
& \times(a_1+a_2+a_3)a_2(a_1+a_3)(a_1+a_3) \\
& \times(a_1+a_2+a_3)(a_1+a_3)(a_1+a_2+a_3) \\
& \times(a_1+a_2+a_3)(a_1+a_2+a_3) \\
& \times(a_1+a_2+a_3)a_2(a_1+a_3) \\
& \times(a_1+a_2+a_3)(a_1+a_3)(a_1+a_2+a_3) \\
& \times(a_1+a_2+a_3)(a_1+a_3)(a_1+a_2+a_3) \\
& \times(a_1+a_2+a_3)(a_1+a_2+a_3)(a_1+a_3) \\
& \times(a_1+a_2+a_3)(a_1+a_2+a_3) \\
= {} & a_2(a_1+a_3) = a_1a_2+a_2a_3.
\end{aligned}
$$

- It means that the decision Table 12 can be reduced and represented in two alternative ways

Table 16

U	a_1	a_2	d
x_1	2	1	1
x_2	3	2	2
x_3	2	1	1
x_4	2	2	2
x_5	1	1	3
x_6	1	1	3
x_7	3	2	2
x_8	1	1	3
x_9	2	1	1
x_{10}	3	2	2

Table 17

U	a_2	a_3	d
x_1	1	3	1
x_2	2	1	2
x_3	1	3	1
x_4	2	3	2
x_5	1	4	3
x_6	1	2	3
x_7	2	1	2
x_8	1	4	3
x_9	1	3	1
x_{10}	2	1	2

- From the *D*-discernibility function, we obtain two Reducts $\{a_1, a_2\}$ and $\{a_2, a_3\}$
- The *D*-discernibility matrix for Reduct $\{a_1, a_2\}$

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

- **Eliminate unnecessary values of condition attribute**

- From the *D*-discernibility function, we obtain two Reducts $\{a_1, a_2\}$ and $\{a_2, a_3\}$
- The *D*-discernibility matrix for Reduct $\{a_1, a_2\}$

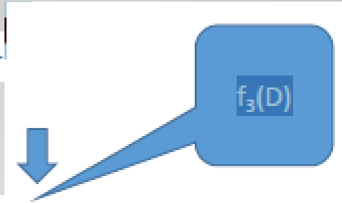

Table 18

	1	2	3	4	5	6	7	8	9	10
1	–	a_1,a_2	–	a_2	a_1	a_1	a_1,a_2	a_1	–	a_1,a_2
2	a_1,a_2	–	a_1,a_2	–	a_1,a_2	a_1,a_2	–	a_1,a_2	a_1,a_2	–
3	–	a_1,a_2	–	a_2	a_1	a_1	a_1,a_2	a_1	–	a_1,a_2
4	a_2	–	a_2	–	a_1,a_2	a_1,a_2	–	a_1,a_2	a_2	–
5	a_1	a_1,a_2	a_1	a_1,a_2	–	–	a_1,a_2	–	a_1	a_1,a_2
6	a_1	a_1,a_2	a_1	a_1,a_2	–	–	a_1,a_2	–	a_1	a_1,a_2
7	a_1,a_2	–	a_1,a_2	–	a_1,a_2	a_1,a_2	–	a_1,a_2	a_1,a_2	–
8	a_1	a_1,a_2	a_1	a_1,a_2	–	–	a_1,a_2	–	a_1	a_1,a_2
9	–	a_1,a_2	–	a_2	a_1	a_1	a_1,a_2	a_1	–	a_1,a_2
10	a_1,a_2	–	a_1,a_2	–	a_1,a_2	a_1,a_2	–	a_1,a_2	a_1,a_2	–

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Eliminate unnecessary values of condition attribute

Discern x1 needs a1 and a2

Discern x6 needs a1 only

$$f_1(D) = (a_1 + a_2)a_2a_1a_1(a_1 + a_2)a_1(a_1 + a_2) = a_1a_2$$

$$f_2(D) = (a_1 + a_2)(a_1 + a_2)(a_1 + a_2)(a_1 + a_2) \times (a_1 + a_2)(a_1 + a_2) = a_1 + a_2$$

$$f_3(D) = (a_1 + a_2)a_2a_1a_1(a_1 + a_2)a_1(a_1 + a_2) = a_1a_2$$

$$f_4(D) = a_2a_2(a_1 + a_2)(a_1 + a_2)(a_1 + a_2)a_2 = a_2$$

$$f_5(D) = a_1(a_1 + a_2)a_1(a_1 + a_2)(a_1 + a_2) \times a_1(a_1 + a_2) = a_1$$

$$f_6(D) = a_1(a_1 + a_2)a_1(a_1 + a_2)(a_1 + a_2) \times a_1(a_1 + a_2) = a_1$$

$$f_7(D) = (a_1 + a_2)(a_1 + a_2)(a_1 + a_2)(a_1 + a_2) \times (a_1 + a_2)(a_1 + a_2) = a_1 + a_2$$

$$f_8(D) = a_1(a_1 + a_2)a_1(a_1 + a_2)(a_1 + a_2) \times a_1(a_1 + a_2) = a_1$$

Table 19

U	a_1	a_2	d
x_1	2	1	1
x_2	*	2	2
x_3	2	1	1
x_4	*	2	2
x_5	1	*	3
x_6	1	*	3
x_7	*	2	2
x_8	1	*	3
x_9	2	1	1
x_{10}	*	2	2

28

*Denotes 'do not care'.

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

Decision rules

$$a_{\mathbf{k}_i} \Rightarrow d_j$$

- Means that "attribute a_k has value i" (If...then...)

Table 19

U	a_1	a_2	d
x_1	2	1	1
x_2	*	2	2
x_3	2	1	1
x_4	*	2	2
x_5	1	*	3
x_6	1	*	3
x_7	*	2	2
x_8	1	*	3
x_9	2	1	1
x_{10}	*	2	2

$$a_{1_2} a_{2_1} \Rightarrow d_1$$

$$a_{2_2} \Rightarrow d_2$$

$$a_{1_1} \Rightarrow d_3$$

Walczak, B.1, Massart, D.L. (1999). Rough sets theory (Tutorial), ***Chemometrics and Intelligent Laboratory Systems*** 47(1): 1–16.

Basic concepts of the rough sets

New Decision

(a) The new object matches exactly one of the deterministic logical rules

(b) The new object matches exactly one of the non-deterministic logical rules

- The rule is ambiguous
- DM is informed about the number of examples which support each rule. The number is called strength
- If the strength of one class is greater than the strength of the other classes occurring in the non-deterministic rule, one can conclude that according to this rule, the considered object most likely belongs to the strongest class.

(c) The new object matches no logical rules

- Outlier or new class

(d) the new object matches more than one logical rule

- If all rules indicate the same decision, there is no ambiguity
- Otherwise, the strength of each rule is determined and DM may treat this case similarly to case (b)

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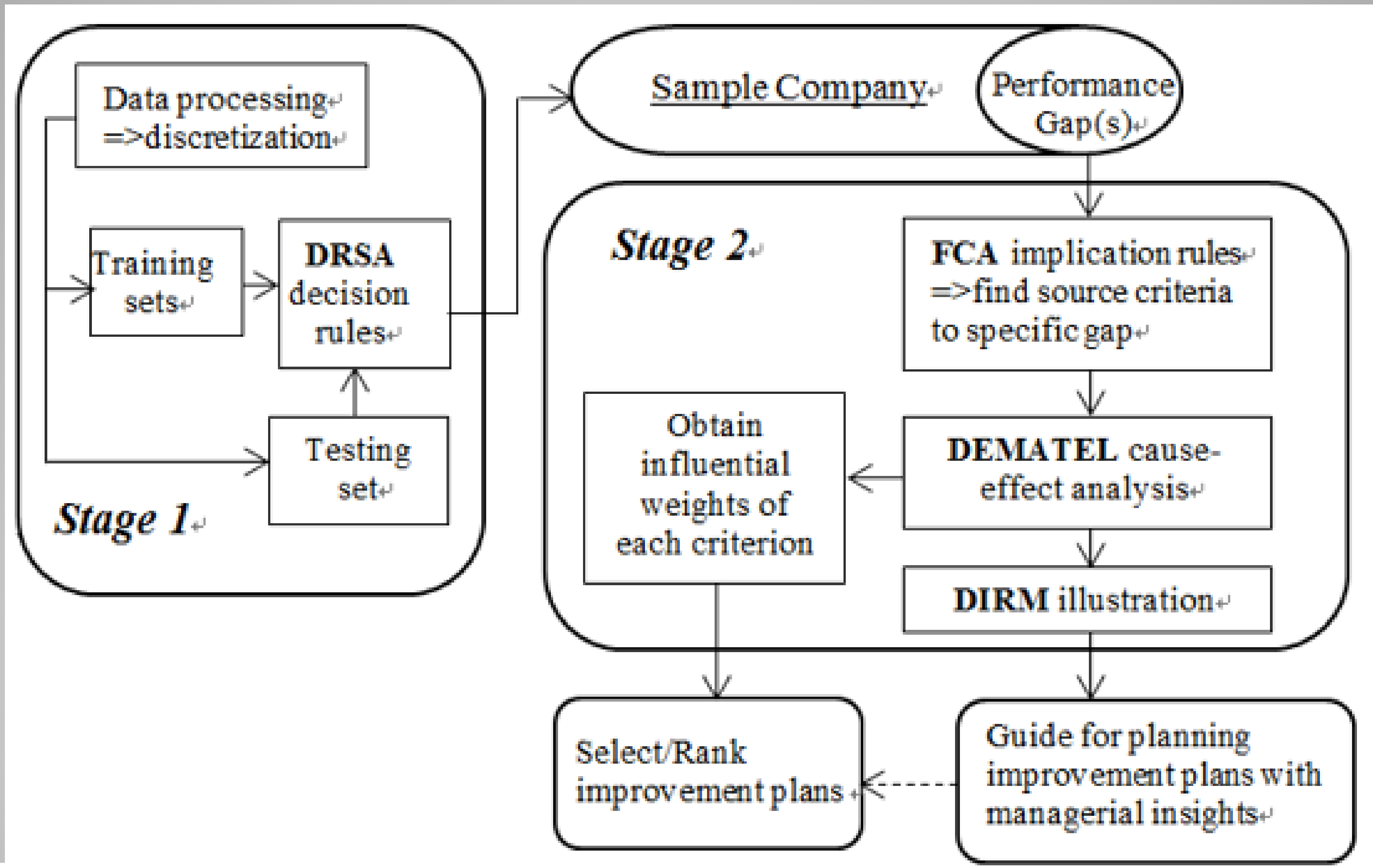
Dominance-based rough set approach (DRSA) MCDM



Two-stage research flows (Guide / Select / Rank FP improvement plans)

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Dominance-based rough set approach (DRSA) MCDM

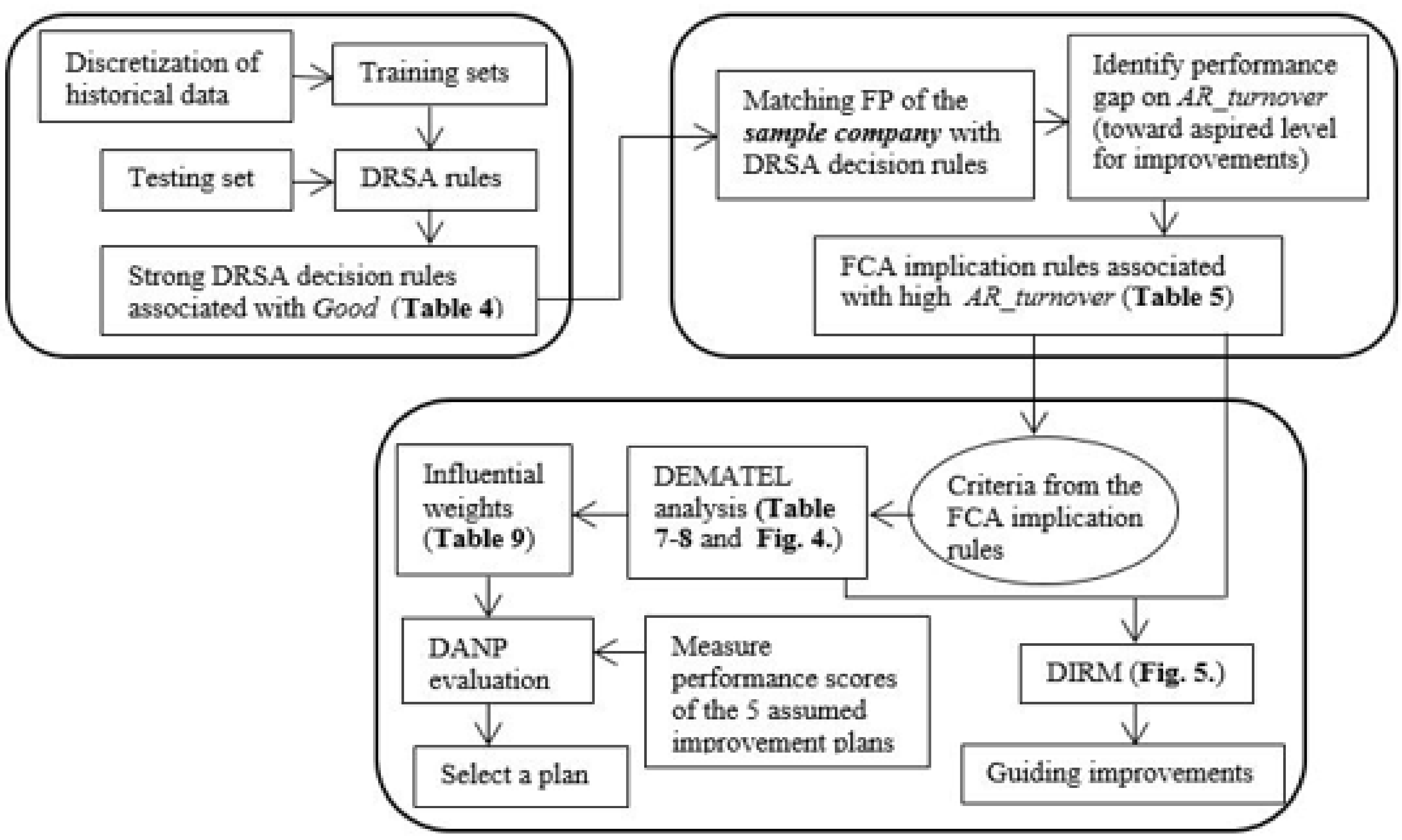

Illustration of research flows of the empirical case

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Dominance-based rough set approach (DRSA) MCDM

Dominance-based Rough Set Approach (DRSA)

- DRSA begins with an information table, and instance (objects) can be placed in rows with attributes in columns.
- Compared with classical RSA (Rough Set Analysis), the main difference of DRSA is the consideration of ordinal evaluation of objects and attributes.
- The typical data table of RS (Rough Set) and DRSA comprises of four tuples, which can be indicated as an information system (IS), for $IS = (U, Q, V, f)$. In the DRSA *IS*, *U* is a finite set of universe, *Q* is a finite set of *k* attributes (i.e., $Q=\{q_1, q_2 \ldots, q_k\}$), V is the value domain of attribute (i.e., $V = \cup_{q \in Q} V_q$), and f denotes a total function (i.e., $f: U \text{ x } Q \rightarrow V$).
- The attributes comprise of condition attributes *C* and decision attribute *D* in a typical DRSA model, and the conditional attributes are often regarded as criteria for a MCDM evaluation problem.

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Dominance-based rough set approach (DRSA) MCDM

Dominance-based Rough Set Approach (DRSA)

- Basic concept rule-process of **Rough Set Theory**: (1) Indiscernibility relation, (2) Lower and upper approximations, (3) Accuracy of approximation, (4) Independence of attributes, (5) Core and reduct of attributes, (6) Core and reducts of attribute values, (7) Classification.
  Reference:
  Walczak, B., Massart, D.L. (1999). Rough set theory (Tutorial). Chemometrics and Intelligent Laboratory Systems, 47(1): 1-16.
- Suppose that there are n objects in *U*, a complete outranking relation on *U* can be defined as $\succeq_q$ with respect to a criterion $q \in Q$; if $x \succeq_q y$ for $x, y \in U$, then it denotes that "x is at least as good as y with respect to criterion q".
- In DRSA, the outranking relation $\succeq_q$ is generally supposed to be a complete preorder relation with respect to criterion *q*.
- Decision attribute $d \in D$ divides *U* into a finite number of decision classes (such as *m* decision classes), i.e., $Cl = \{Cl_t : Cl_1, Cl_2, \ldots, Cl_m\}$ for *t*=1,2,…,*m*.

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Dominance-based rough set approach (DRSA) MCDM

(1)

(2)

Dominance-based Rough Set Approach (DRSA)

- For each $x \in U$, object x belongs to only one class Cl_t ($Cl_t \in Cl$). Assume that *Cl* has preferential order (i.e., for all $r, s = 1, \ldots, m$, if $r \succ s$, the decision class Cl_r is preferred to Cl_s), an downward union $Cl_t^{\leq}$ and upward union $Cl_t^{\geq}$ of classes can be defined as Eq. (1)-(2):

$$Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s \quad \cdots\cdots\cdots\cdots \quad (1)$$

$$Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s \quad \cdots\cdots\cdots\cdots \quad (2)$$

- The upward union is used in this study to identify the good decision class (i.e., positive FP change in the next period); therefore, only the upward union of classes is discussed hereafter. The condition attributes (criteria) can be used to classify decision classes by dominance relations. Given a set of attributes $P \subseteq C$ and $x, y \in U$, x dominates y with respect to set of attributes *P* could be denoted by $xD_P y$ to represent x *P*-dominates y.

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Dominance-based rough set approach (DRSA) MCDM

Dominance-based Rough Set Approach (DRSA)

- Therefore, a set of objects (instances) dominating x is termed as P-dominating set in Eq. (3), and a set of objects dominated by x is called P-dominated set in Eq. (4):

$$D_P^+(x) = \{y \in U : yD_P x\} \quad (3)$$

$$D_P^-(x) = \{y \in U : xD_P y\} \quad (4)$$

- The P-dominating set and P-dominated set can be used to representing a collection of upward and downward unions of decision classes, which may represent granules of knowledge. The P-lower and P-upper approximation of an upward union with respect to $P \subseteq C$ can be define by Eq. (5) and Eq. (6) respectively:

$$\underline{P}\left(Cl_t^{\geq}\right) = \left\{x \in U : D_P^+(x) \subseteq Cl_t^{\geq}\right\} \quad (5)$$

$$\overline{P}(Cl_t^{\geq}) = \left\{x \in U : D_P^- \cap Cl_t^{\geq} \neq \varnothing\right\} \quad (6)$$

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Dominance-based rough set approach (DRSA) MCDM

Dominance-based Rough Set Approach (DRSA)

- The P-lower approximation $\underline{P}\left(Cl_t^{\geq}\right)$ denotes all of the objects $x \in U$ that are for sure to be included in the upward union $Cl_t^{\geq}$, whereas all objects have at least the same or better evaluation with regard to all criteria $P \subseteq C$. With the P-upper approximation and P-lower approximation of $Cl_t^{\geq}$, the P-boundary of $Cl_t^{\geq}$ is defined as Eq. (7):

$$Bn_P = \overline{P}(Cl_t^{\geq}) - \underline{P}\left(Cl_t^{\geq}\right) \quad (7)$$

- The so-called dominance principle requires that if an object x dominating object y on all considered criteria $P \subseteq C$ (i.e., in conditional part), then the object x should also dominate y on the decision attribute. The objects that comply with the dominance principle are called consistent; otherwise, inconsistent. Moreover, the quality of approximation is defined as the ratio in Eq. (8), and the ratio $\gamma_P(Cl)$ can be regarded as a consistency ratio, for all the objects from U and all considered condition attributes $P \subseteq C$.

$$\gamma_P(Cl) = \frac{\left| U - \left(\bigcup_{t \in \{2,\ldots,n\}} Bn_P\left(Cl_t^{\geq}\right) \right) \right|}{|U|} \quad (8)$$

Dominance-based rough set approach (DRSA) MCDM

- Furthermore, the accuracy of approximation of ordered classes $Cl_t^{\geq}$ with regard to a set of criteria $P \subseteq C$ is defined as $\alpha_P\left(Cl_t^{\geq}\right)$ in Eq. (9), and $|\bullet|$ in Eq. (8)-(9) is the cardinality of a set.

$$\alpha_P\left(Cl_t^{\geq}\right) = \frac{\left|\underline{P}\left(Cl_t^{\geq}\right)\right|}{\left|\overline{P}(Cl_t^{\geq})\right|} \quad (9)$$

- Each minimal subset $P \subseteq C$ that may satisfy $\gamma_P(Cl) = \gamma_C(Cl)$ is called a REDUCT of Cl, and the intersection of all REDUCTs represent the indispensable attributes to maintain the quality of approximation, called CORE$_{Cl}$. Using the dominance-based approximation approach, a set of decision rules can be obtained in the form of "**if** *antecedent* **then** *consequence*", which can support DMs to identify a company's performance gaps on the critical criteria (financial indicators) in decision rules, to plan for improvements for its FP.

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Dominance-based rough set approach (DRSA) MCDM

Dominance-based Rough Set Approach (DRSA)

- The DRSA decision rules comprise of two types: certain and possible; the certain decision rules provide conditions for objects belonging to $\underline{P}(Cl_t^{\geq})$, mainly used in this study. The details of DRSA can be found in (Greco *et al.* 2001; Greco *et al.* 2002; Błaszczyński *et al.* 2007; Błaszczyński *et al.* 2013). To conduct DRSA modeling in this study, the required steps are as below:

Step 1: Define condition attributes and decision attribute of a semiconductor stock, and conduct a three-level discretization for all the attributes, i.e., including condition attributes and decision attribute. The three-level discretization may deliver more intuitive understanding for DMs to interpret obtained decision rules by comparing the relative performance of a company with its peer group on each criterion. The details of the used three-level discretization in this study will be explained in Subsection 3.1.

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Dominance-based rough set approach (DRSA) MCDM

Dominance-based Rough Set Approach (DRSA)

Step 2: Match the values of a stock's condition attributes in time period (t-1) with its decision class in time period t to denote an object (instance), and the matched data set is devised to predict the FP of a stock in the subsequent period by using its current financial data.

Step 3: Construct DRSA model and obtain decision rules to identify stocks with plausible good FP in the next period. The validation of DRSA model will be further illustrated in Section 3.

- **Formal concept analysis (FCA)**

- Originated from applied mathematics, FCA was developed based on mathematical order and lattice theory, which has been applied in various fields, such as software engineering, knowledge acquisition, medical classification, and financial investment.
- FCA can be defined as a set of structure $\Re := (G, M, I)$, and I denotes the binary relation between two sets: G and M. The elements in the set G represent objects, and the elements in the set M denote attributes.

Dominance-based rough set approach (DRSA) MCDM

- Thus, a formal context can be formed by connecting the objects in G to attributes in M through the binary relation I (yes or no), i.e., $(g,m)\in I$ for $g\subseteq G$ and $m\subseteq M$. If $g=m^{I}$ and $m=g^{I}$; then g and m can be called the extent and intent of a pair of formal concept (g,m). Based on the theorem in concept lattice (Ganter *et al.* 1997), while the concept lattice of (G,M,I) is a complete lattice, it should be made up of the closed subsets (i.e., sub-lattices).
- The closed subset property provides the foundation for calculating Duquenne-Guigues base of implications, which has a minimal number of implication rules. In this study, the identified performance gap on a certain criterion can be regarded as a m in the attribute set M, and the Duquenne-Guigues implication rules (Ganter *et al.* 1997; Wille 2005) can be obtained to explore the extents with high object supports.

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Dominance-based rough set approach (DRSA) MCDM

Step 4: Examine a target company's performance on the strong decision rules (associated with good FP change in the subsequent period), and identify the top performance gap.

Step 5: Conduct Duquenne-Guigues implication reasoning in FCA to obtain implication rules associated with the source criteria that might lead to the identified performance gap attribute in **Step 4**.

- With FCA implication analysis, decision makers could have a guidance regarding the source factors (criteria) related to the underperformed criterion (identified by DRSA decision rules) for a company. In a real business environment, the criteria regarding a company's FP are often interrelated; the FCA is proposed to induct from positive alternatives in the historical data, for finding the source criteria of the underperformed criterion for an individual company.

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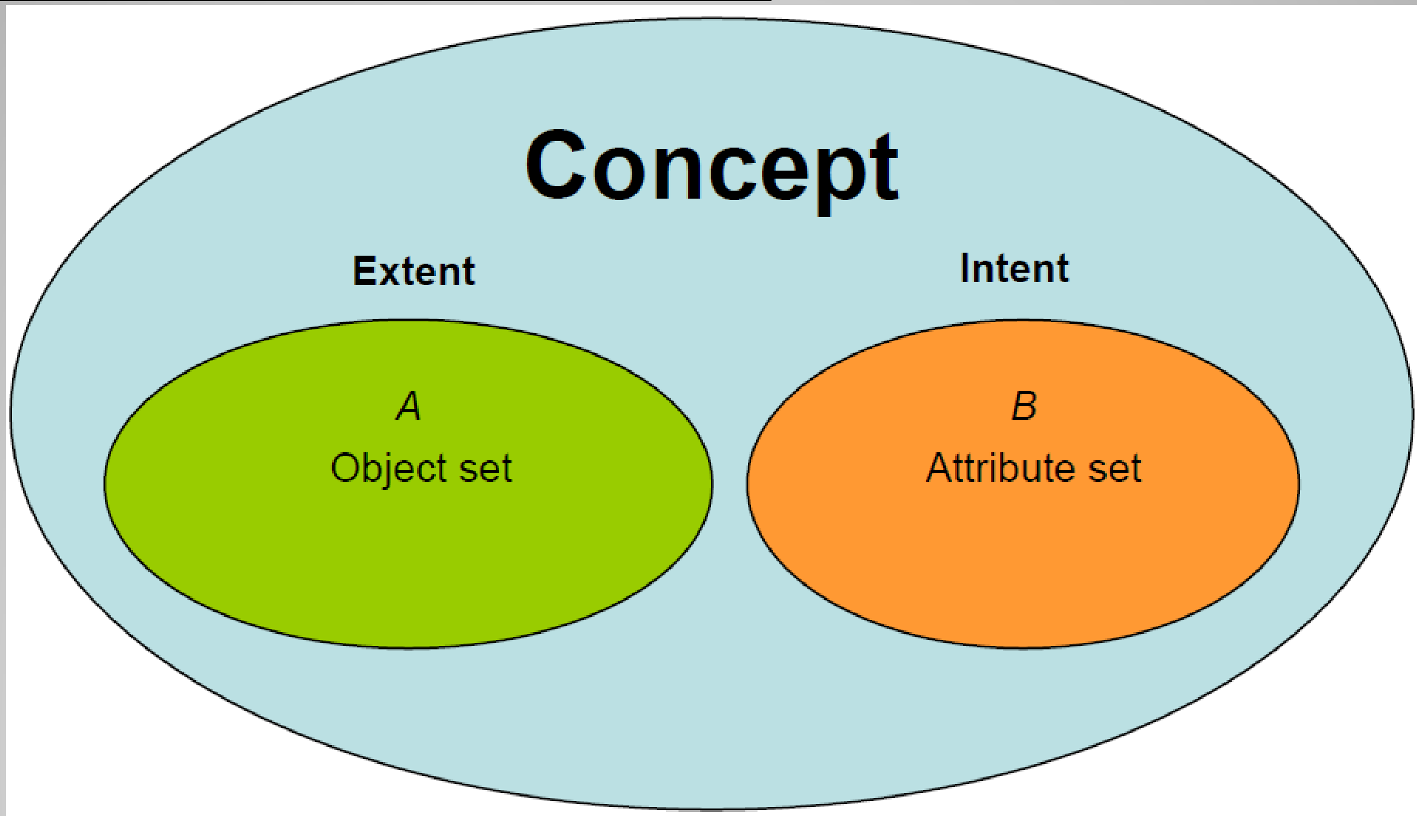
Dominance-based rough set approach (DRSA) MCDM – Formal Concept Analysis

Formal Concept Analysis



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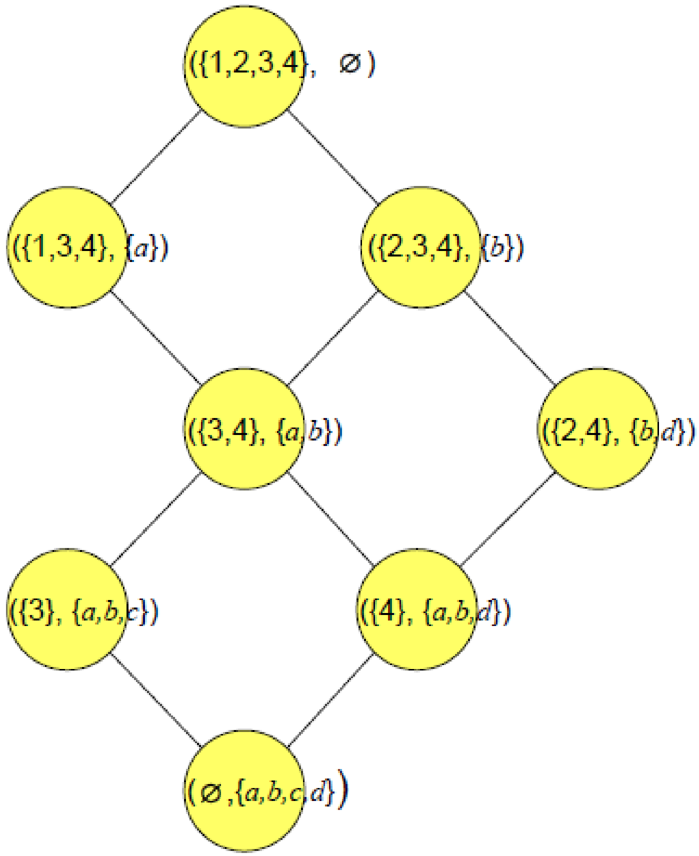
Dominance-based rough set approach (DRSA) MCDM – Formal Concept Analysis

Formal Concept Analysis

Formal context

Objects \ Attributes	*a*	*b*	*c*	*d*
1	v			
2		v		v
3	v	v	v	
4	v	v		v



Complete lattice

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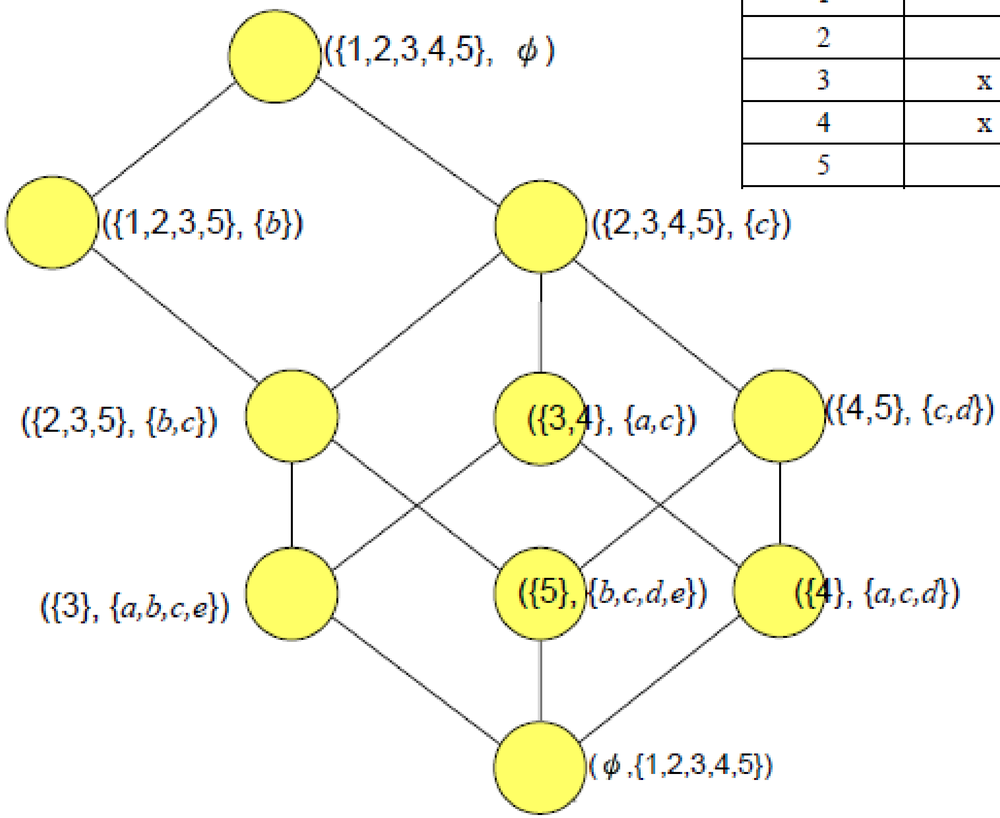
Dominance-based rough set approach (DRSA) MCDM – Formal Concept Analysis

Formal Concept Analysis

	a	*b*	*c*	*d*	*f*
1		x			
2		x	x		x
3	x	x	x		x
4	x		x	x	
5		x	x	x	x



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Dominance-based rough set approach (DRSA) MCDM – Formal Concept Analysis

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References for Formal Concept Analysis

New concepts and trends of hybrid MCDM model for Tomorrow: Some examples for the real cases

- Dominance-based rough set approach (DRSA) MCDM
- MADM: DEMATEL, DANP (DEMATEL-based ANP), Integration (Additive: SAW, VIKOR, Grey Relation Analysis, PROMETHEE, ELECTRE; Non-additive: Fuzzy Integral)
- MODM: Changeable Spaces Programming

MADM (Multiple Attribute Decision Making

Tzeng classify **MCDM problems** into three main categories: **multiple rule/-based decision making (MRDM), multiple attribute decision making (MADM**), and **multiple objective decision making (MODM))** based on the different purposes and the different data types in interrelationship**. MADM** applied in the **evaluation for ranking, selection**, and **improvement** which usually associated with a limited number of predetermined alternatives and the discrete preference ratings in **interdependent problems**. **MODM** is especially **suitable for the design/planning**, which is to achieve the **best or called aspired goals** (aspiration level) by considering the various interactions within the given constrains, how relax or relieve the given constrains through innovation and creativity so **that both decision and objective spaces are changeable in new concepts of our research.**

MADM (Multiple Attribute Decision Making

Basic concepts

- A typical **MADM** is a scientific analytical method for evaluating a set of **criteria/attributes** and **alternatives** based on considering a set of **multiple,** i.e., data set of information systems as, $IS = (U, A, V, f)$.
- However, we find that the traditional MADM ignored **some important new concepts** and **have some assumptions/ hypothesis limit/defects for solving real-world** problems; for example, many traditional **Economics and Statistics are unrealistic of assumption in the real world**, such as assuming independent problem, using coefficients of correlation (not measuring influential relationship among criteria), linear (e.g., multi-regression, Econometric), etc.

MADM (Multiple Attribute Decision Making (Basic concepts)

MADM

- **First**, combining **multiple cause-effect rule-based decision** with DEMATEL technique of INRM (or called DIRM (directional-influential relation map) flow graph) and **hybrid modified DANP-MADM** for performance improvement
- **Second,** the traditional model assumes criteria are **independent** with hierarchical structure; but the **relationships** between criteria or dimensions are usually **interdependent** and sometimes even exit **feedback** effects in the real-world.
- **Third,** the relative good solution from the existing alternatives is replaced by the **aspiration levels** to fit today's competitive markets.
- **Fourth**, the trends have shifted from how can be "ranking" or "selection" the most preferable alternatives to how can be **"improvement**" their performances.
- **Fifth**, information fusion/aggregation such as fuzzy integral, a **non-additive/super-additive model**, has been developed to aggregate the performances.

Some examples for the real cases: New hybrid MCDM model

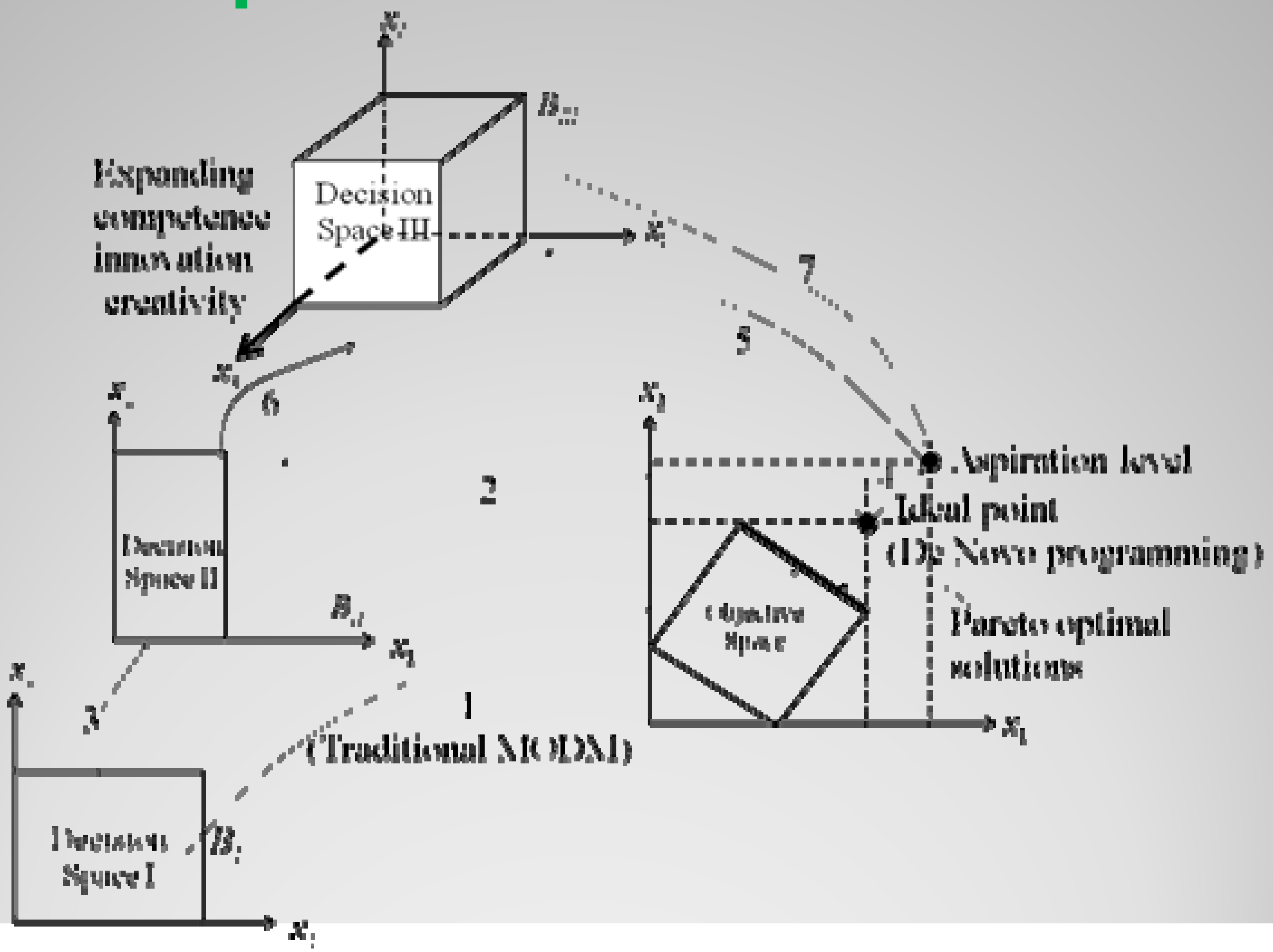
Basic concepts

- **Why we don't use "traditional Statistics and Economics" approaches:** Traditional Statistics and Economics are unrealistic in the real world.
- **Setting aspiration level**: For avoiding "Choose the best among inferior choices", i.e., for avoiding "Pick the best apple among a barrel of rotten apples".
- **Constructing influential network relation map (INRM) for systematic improvement**: We need to find a cure to the problem instead of just treating its symptoms; i.e., we need a systematic approach to problem-solving. Instead of addressing the symptoms of the problem, we need to identify the sources of the problem.

Some examples for the real cases: New hybrid MCDM model

Basic concepts

MODM



The concept of changeable decision space and aspiration level

MADM (Multiple Attribute Decision Making (Basic concepts)

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Purposes of new hybrid MADM methods

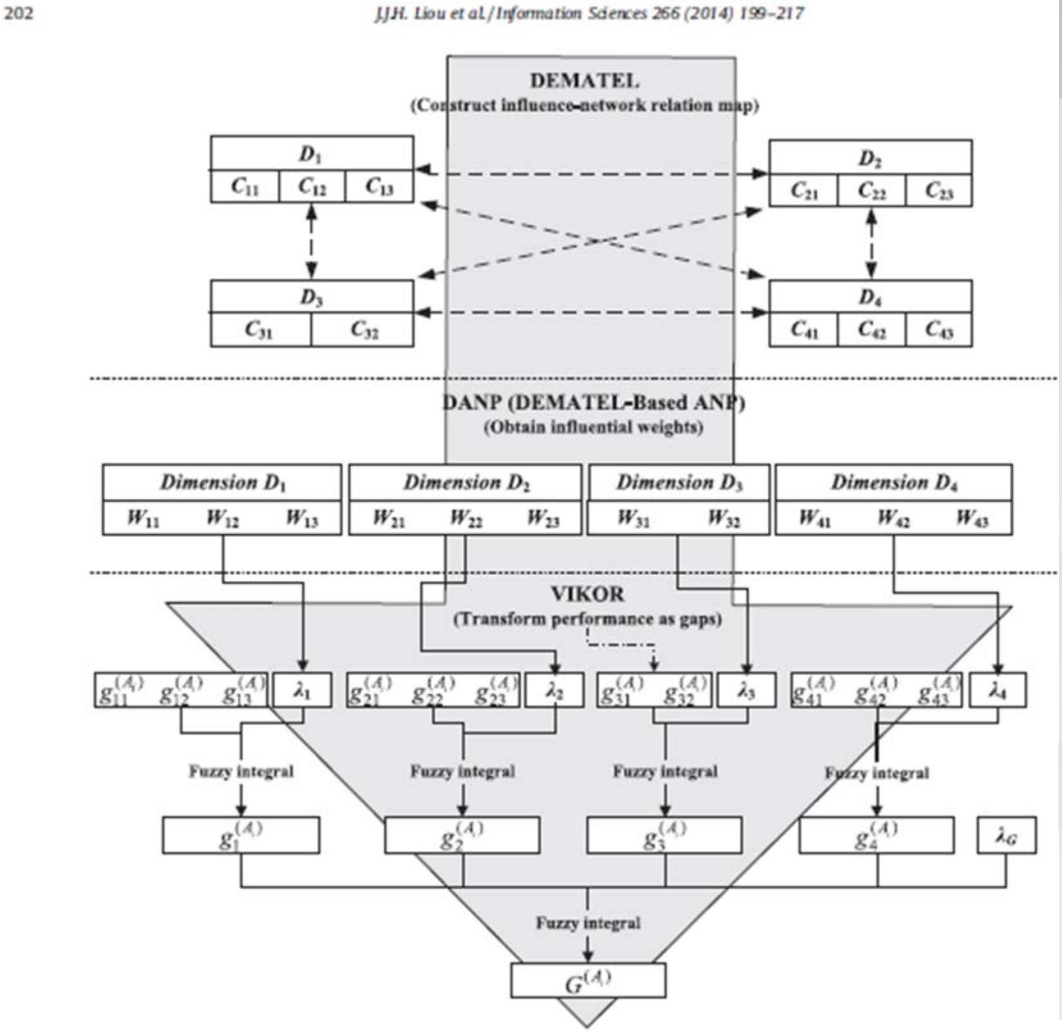
The purposes of our proposed these new hybrid MADM methods:

- Not only in order to overcome the defects of conventional MADM method, we have focused on developing **a series of new Hybrid Dynamic Multiple Attribute Decision Making** (HDMADM) method for solving the complication dynamic problem in real world and applying to various fields.
- But also in order **to: (1) avoid "Statistics and economics are unrealistic in the real world"**; **(2) avoid** "**choose the best among inferior choices/ options/alternatives**, i.e., **avoid "Pick the best apple among a barrel of rotten apples"; (3) deal with super-additive/non-additive problems in the real world; (4)** "**we need a systematic improvement, we need to identify the sources of the problem, i.e., avoid "stop-gap piecemeal (腳痛醫腳頭痛醫頭)**" **for achieving aspiration level** in each criterion**.** Finally empirical real cases are illustrated to **and effectiveness** of the proposed **new hybrid MADM methods for solving the real world problems**.

Concept of Methods

- **DEMATEL** technique is used to construct the cause-effects of interactions/interrelationship between criteria (called influence matrix) and build an influential network relation map (INRM).
- **DANP** (DEMATEL-based ANP) for deriving global influential weights (for solving interdependence and feedback dynamic problems)
- **VIKOR** uses the class distance function (Yu, 1973), based on the concept how can be closest to positive-ideal (the Aspiration level) solution and furthest away from the negative-ideal (the Worst level) solution for improvement the gaps of each criterion (different from max-min approach in tradition in order).
- **Fuzzy integral** for integrating the performance value (fusing information in performance matrix) of value function (non-additive/super-additive approach), i.e., one plus one is larger than two (1+1 > 2).

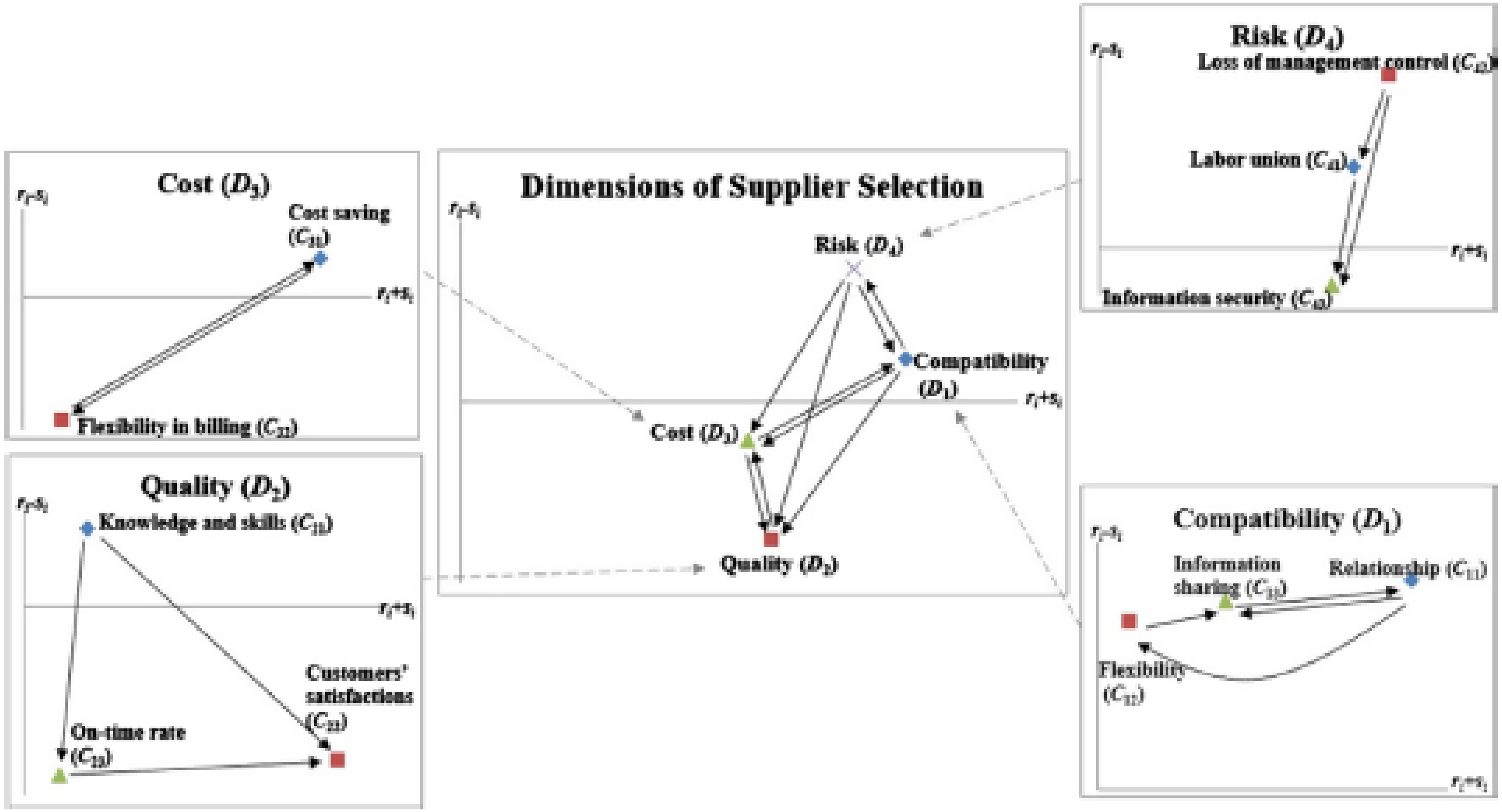
Basic Concepts of New Hybrid MADM Model



Improvement

James J.H. Liou, Yen-Ching Chuang, Gwo-Hshiung Tzeng (Corresponding author) (2013) "A fuzzy integral-based model for supplier evaluation and improvement, **Information Sciences, 266**, 199–217 (Impact factor: **3.643**, 5-Year Impact Factor: **3.676, 2012**).

Example in the real world



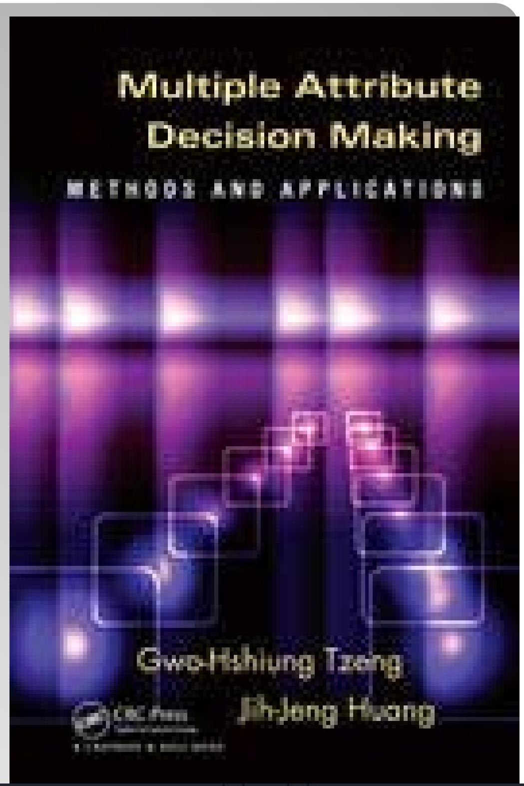
James J.H. Liou, Yen-Ching Chuang, Gwo-Hshiung Tzeng (Corresponding author) (2013) "A fuzzy integral-based model for supplier evaluation and improvement, Information Sciences, 266, 199–217 (Impact factor: 3.643, 5-Year Impact Factor: 3.676, 2012).

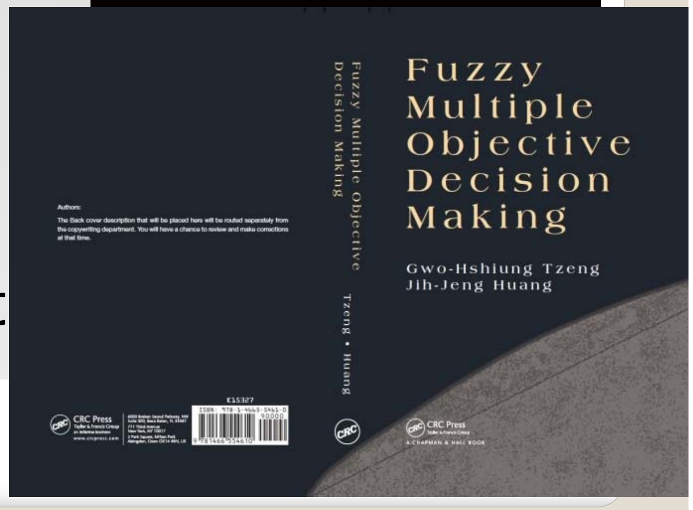
Research Methods for Problems-Solving

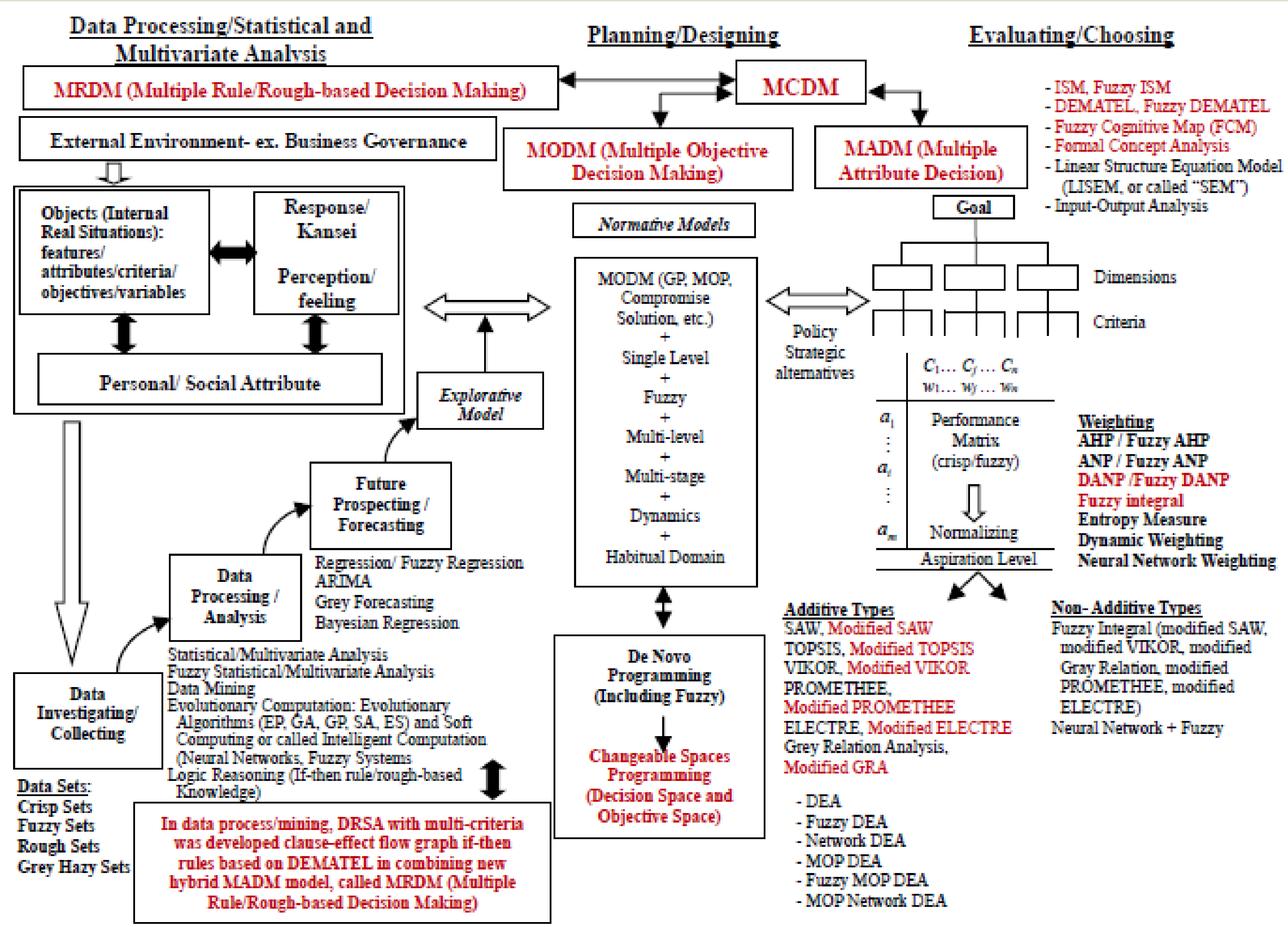
- Rough sets (DRSA), DEMATEL
- ANP
- DANP (DEMATEL-based ANP)
- VIKOR, Grey Relation Analysis, PROMETHEE, etc.
- Fuzzy Integral (Non-additive/ Super-additive)
- Improvement by changeable spaces MOP programming

→ Hybrid MCDM Methods

For Problems-solving - Improvement



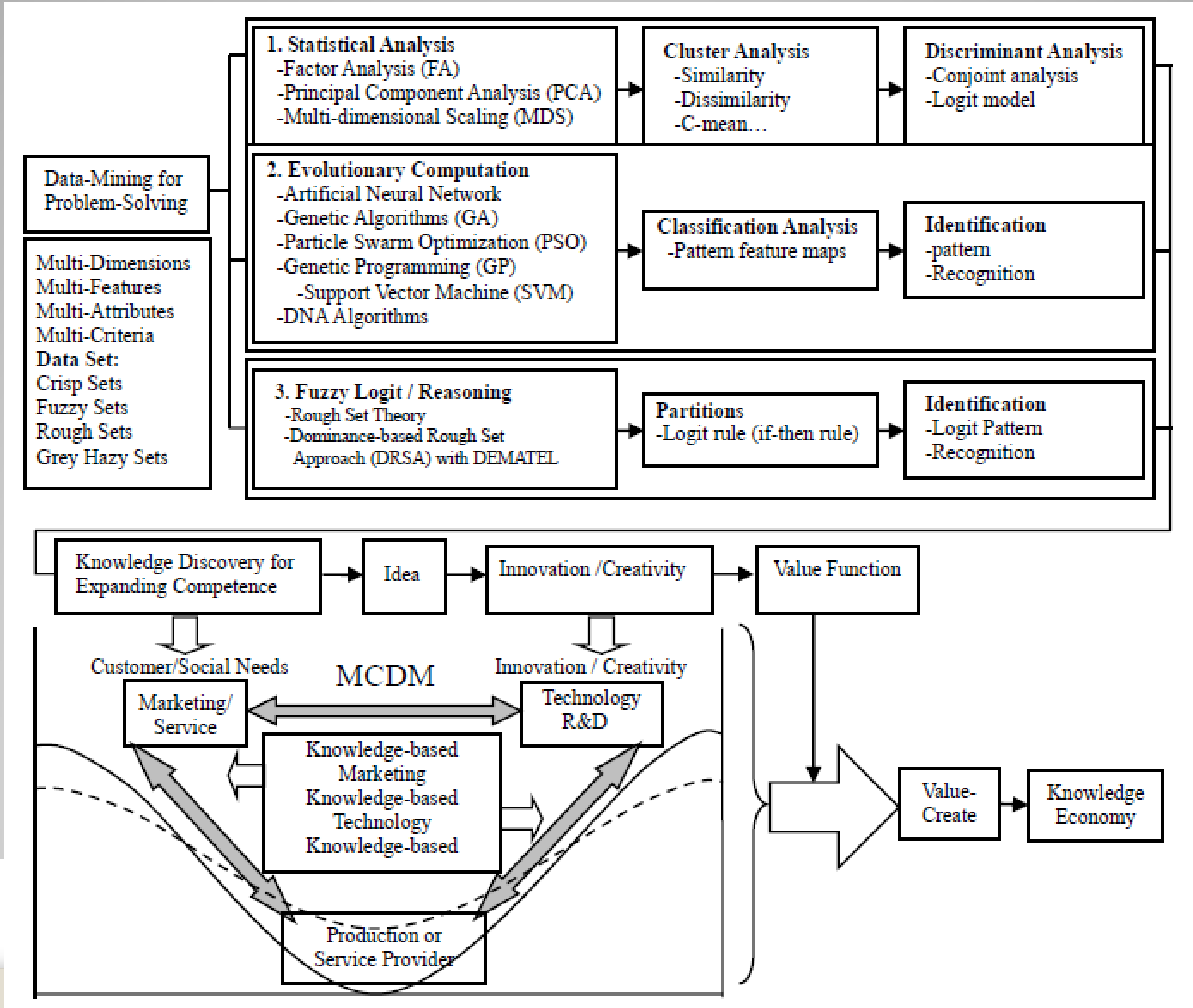




The basic concepts and framework in 'Research Methods for Problem Solving'

(Tzeng and Huang, 2011, 2013; Liou and Tzeng, 2012; Peng and Tzeng, 2013)

Data Mining Concepts of Intelligent Computation in Knowledge Economy



Background -A Quick Overview of Traditional MCDM Approaches

- Criteria weight calculations by AHP (assuming criteria independences) or
- ANP based weight derivations by a decision problem structure being derived arbitrarily (based on assumption, Saaty)
- TOPSIS which determines a solution with
  - The shortest distance from the ideal solution and
  - The farthest distance from the negative-ideal solution (cannot be used for **ranking purpose**)

Opricovic, S., Tzeng, G.H. (2004). **Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPISIS**, *European Journal of Operational Research*, *Volume 156*, *Issue 2*, *16 July 2004*, *Pages 445-455*
 (***Essential Science Indicators***^{sm} to be one of the **most cited papers** in the **field of Economics**).

Background - Problems being Faced by Traditional MCDM Approaches

Alternatives being derived as is

- Wrong assumptions on the **independences** between the determinants (very few exists in the real world)
- Vague correlations between criteria, such as, SEM, etc., improved by using DEMATEL technique ("**Statistics and Economics are unrealistic in the real world**", using independent, additive, and so on problems).
- The lack of improvement of each alternative (improvement is more important, **avoid "stop-gap piecemeal (腳痛醫腳頭痛醫頭)"**..
- Compromise solutions being derived (e.g. by TOPSIS) is not always the closest to the ideal (cannot be used for **ranking purpose**)
- "Rotten (decay, not good) apples versus rotten apples" situation

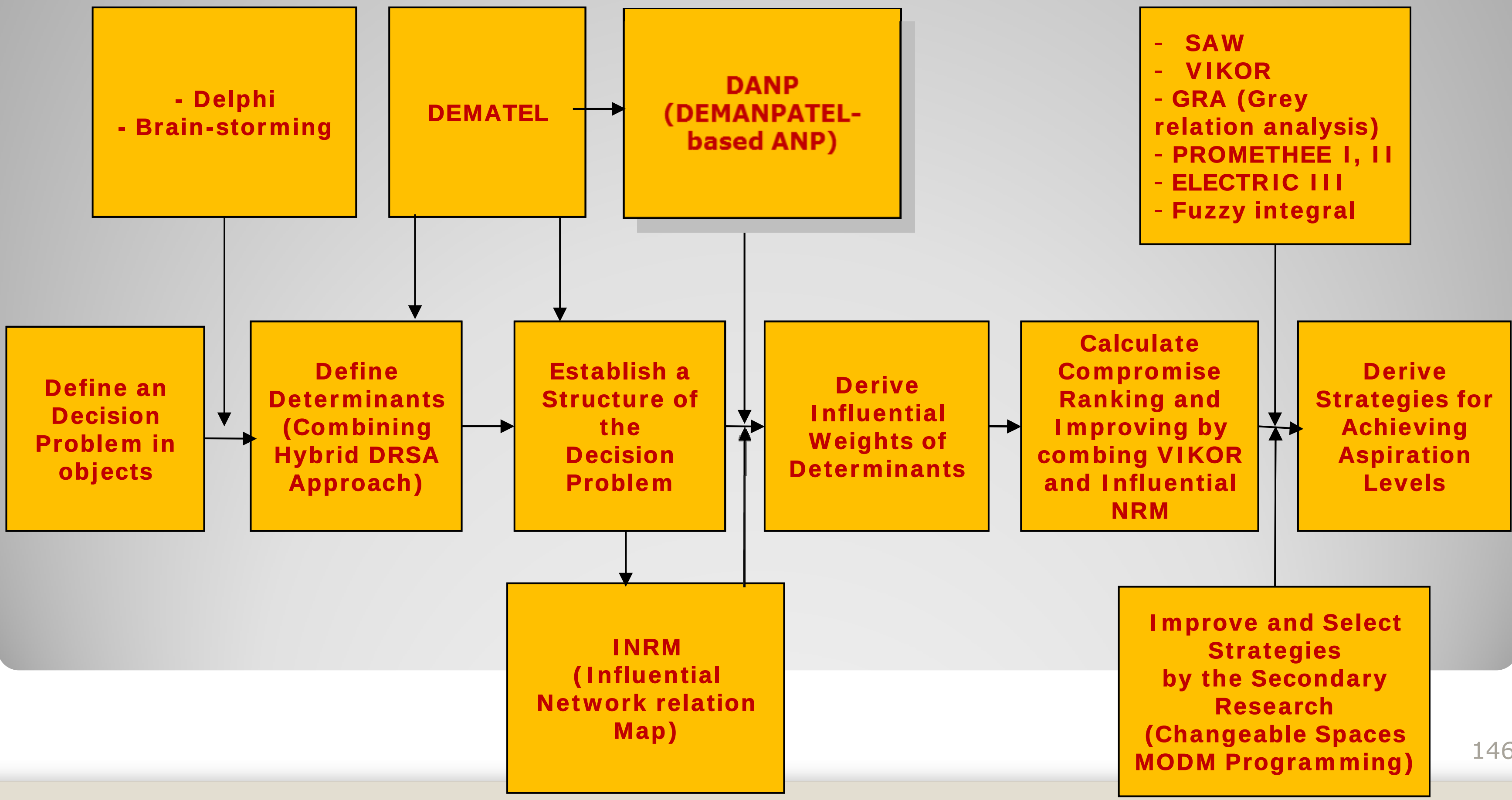
Purpose

- Introduce for solving the suitable real world MCDM problems, and the above mentioned problems should be corrected
  - A proposal of new concepts and trends of novel hybrid MCDM framework is essential in my two new books and in my publication papers of our research group

Appreciate I have an opportunity to talk **"New concepts and trends of hybrid MCDM model for tomorrow"** including my two new books and a series of recent published SSCI/SCI journal papers for sharing with our Colleagues of **National Taipei University** for **solving actual/real world problems in business and economics** by Academic Speaker in this talk..

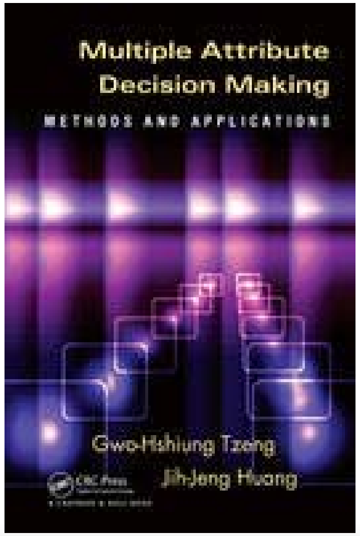
Research Methods

Combined DEMATEL Technique with a Hybrid Novel MCDM Method for applying the real case



DEMATEL - Decision Making Trial and Evaluation Laboratory

New Methods



Basic Concept (1)

- The DEMATEL method was developed by the Battelle Geneva Institute to
  - Analyze complex "world problems" dealing mainly with interactive man-model techniques in complex social systems (Gabus and Fontela, 1972) for improving traditional "System Dynamics" by Forester" (in 1960-1970s), then we use this basic concepts for using to evaluate qualitative and factor-linked aspects of social problems by natural language.
  - We, also based on these concepts, develop a series of novel hybrid MADM model, such as Liou et al. (2007), Tzeng et al. (2007); Ou Yang, et al. (2008), Liu et al. (2012) and so on.
- **The applicability of the method can be widespread**
  - Industrial planning and improvement
  - Decision-making to transportation planning, urban planning and design
  - Regional environmental assessment
  - Analysis of world problems
  - Social network analysis, and
  - Others

Basic Concept (2)

- The DEMATEL method is based upon graph theory
  - Enabling us to plan and solve complex problems visually
    - We may divide multiple criteria into a cause-and-effects group, in order to better understand causal relationships and build influential network relationship map (INRM) in interdependence and feedback problems for improving the gaps of criteria to achieve **aspiration levels** in **satisfaction** [Solving and treating the basic concepts proposed by **Herbert Simon**, 1978 **Nobel Prize**].

Relation Graphs (1)

- Directed, in-directed, and total relation graphs (also called digraphs) are more useful than directionless graphs
  - Digraphs (such as **SEM** model etc.) will demonstrate the directed, in-directed and total relationships of sub-systems, but based on Hypotheses.
- A digraph typically represents a communication network, or a domination relationship between individuals, etc.
- Suppose a system contains a set of elements, $S = \{s_1, s_2, \ldots, s_n\}$, and particular pair-wise relationships are determined for modeling, with respect to a mathematical relationship, **MR**.

Relation Graphs (2)

- Next, portray the influential relationship (RG) as a influence matrix that is indexed equally in both dimensions by elements from the set *S* by directed relation graph. Then, extract the case for which the number 0 (completely no influence) to 4 (extremely or very high influence) appears in the cell (i,j) by directed relation graph, if the entry is a positive integral that has the meaning of:
  - the ordered pair (s_i, s_j) is in the relationship;
  - it has the kind of relationship regarding that element such that s_i causes element s_j.

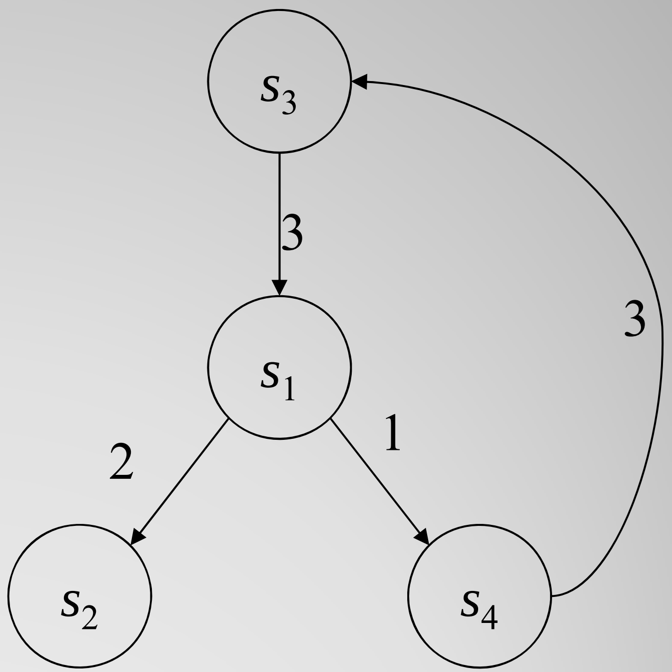
Relation Graphs (3)

- The number between factors is influence or influenced degree.
- The DEMATEL method can convert the relationship between the causes and effects of criteria into an intelligible structural model of the system

Relation Graphs (4)

Directed Relation Graph

- The elements, S_1, S_2, S_3 and S_4 represent the factors that have relationships in the digraph.
- The number between factors is influence or influenced degree.
  - For example, an arrow from S_1 to S_2 represents the fact that influences and its influenced degree is two.



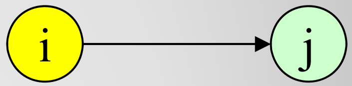
Definitions (1)

- Definition 1
  - The pair-wise comparison scale may be designated as eleven levels, where the scores, such as ‘completely no influence (0),’ ‘low influence (1),’ ‘medium influence (2),’ ‘high influence (3),’ and ‘very high influence (4),’ respectively (or 0, 1, 2, 3, 4 or 0, 1, 2,..., 10) represent the range from ‘no influence’ to ‘very high influence’.

Definitions (2)

- Definition 2
  - The initial direct relation/influence matrix $\boldsymbol{A}$ is an $n \times n$ matrix obtained by pair-wise comparisons, in terms of influences and directions between the criteria, in which a_{ij} is denoted as the degree to which the i^{th} criteria affects the j^{th} criteria.

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & a_{ij} & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$



Definitions (3)

- Definition 3
  - The normalized direct relation/influence matrix $\boldsymbol{X}$ can be obtained through Equations (1) and (2) by normlization, in which all principal diagonal elements are equal to zero.

$$\boldsymbol{N} = s\boldsymbol{A} \qquad (1)$$

where

$$s = 1 \Big/ \max\left\{ \max_{1\le i\le n} \sum_{j=1}^{n} a_{ij}, \max_{1\le j\le n} \sum_{i=1}^{n} a_{ij} \right\} \qquad (2)$$

$$\left(\text{or } s = \min\left\{ 1 \Big/ \max_{1\le i\le n} \sum_{j=1}^{n} a_{ij}, 1 \Big/ \max_{1\le j\le n} \sum_{i=1}^{n} a_{ij} \right\} \right)$$

In this case, $\boldsymbol{X}$ is called the normalized matrix.

Since $\lim_{g\to\infty} \boldsymbol{X}^{g} = [0]$

Definitions (4)

- Definition 4
  - Then, the total relationship matrix $\boldsymbol{T}$ can be obtained using Equation (3), where $\boldsymbol{I}$ stands for the identity matrix.
  - $$\boldsymbol{T} = \boldsymbol{X} + \boldsymbol{X}^2 + \ldots + \boldsymbol{X}^g$$
  - $$= \boldsymbol{X}\left(\boldsymbol{I} + \boldsymbol{X} + \ldots + \boldsymbol{X}^{g-1}\right)[(\boldsymbol{I} - \boldsymbol{X})(\boldsymbol{I} - \boldsymbol{X})^{-1}]$$
  - $$= \boldsymbol{X}\left(\boldsymbol{I} - \boldsymbol{X}^g\right)(\boldsymbol{I} - \boldsymbol{X})^{-1}$$
    then, $\boldsymbol{T} = \boldsymbol{X}(\boldsymbol{I} - \boldsymbol{X})^{-1}$, $\quad \lim_{g \to \infty} \boldsymbol{X}^g = [0]$ when $g \to \infty \qquad (3)$
  - where $\boldsymbol{X} = [x_{ij}]_{n \times n}$, $0 \leq x_{ij} < 1$, $0 < \sum_{j=1}^{n} x_{ij} \leq 1$ and $0 < \sum_{i=1}^{n} x_{ij} \leq 1$,
  - If at least one row or column of summation, but not all, is equal to 1, then $\lim_{g \to \infty} \boldsymbol{X}^g = [0]$ and $\boldsymbol{T}$ is a total influence-related matrix; matrix $\boldsymbol{X}$ is a direct influence matrix and
  - matrix $\left(\boldsymbol{X} + \boldsymbol{X}^2 + \ldots + \boldsymbol{X}^g\right)$ stands for a indirect influence matrix. The (i,j) element t_{ij} of matrix $\boldsymbol{T}$ denotes the direct and indirect influences of factor i on factor j.

$\boldsymbol{T} = [t_{ij}], \quad i, j \in \{1, 2, \ldots, n\}$

Definition (5)

- Definition 5
  - The row and column sums are separately denoted as vector $\boldsymbol{r}$ and vector $\boldsymbol{c}$ within the total-relation matrix $\boldsymbol{T}$ through Equations (4), (5), and (6).

$$\boldsymbol{T} = [t_{ij}], \quad i, j \in \{1, 2, \ldots, n\} \tag{4}$$

$$\boldsymbol{r} = [r_i]_{n\times 1} = \left[\sum_{j=1}^{n} t_{ij}\right]_{n\times 1} = (r_1, \ldots, r_i, \ldots, r_n)' \tag{5}$$

$$\boldsymbol{d} = [d_j]_{n\times 1} = \left[\sum_{i=1}^{n} t_{ij}\right]'_{1\times n} = (d_1, \ldots, d_j, \ldots, d_n)' \tag{6}$$

where the vector $\boldsymbol{r}$ and vector $\boldsymbol{d}$ vectors denote the sums of the rows and columns, respectively.

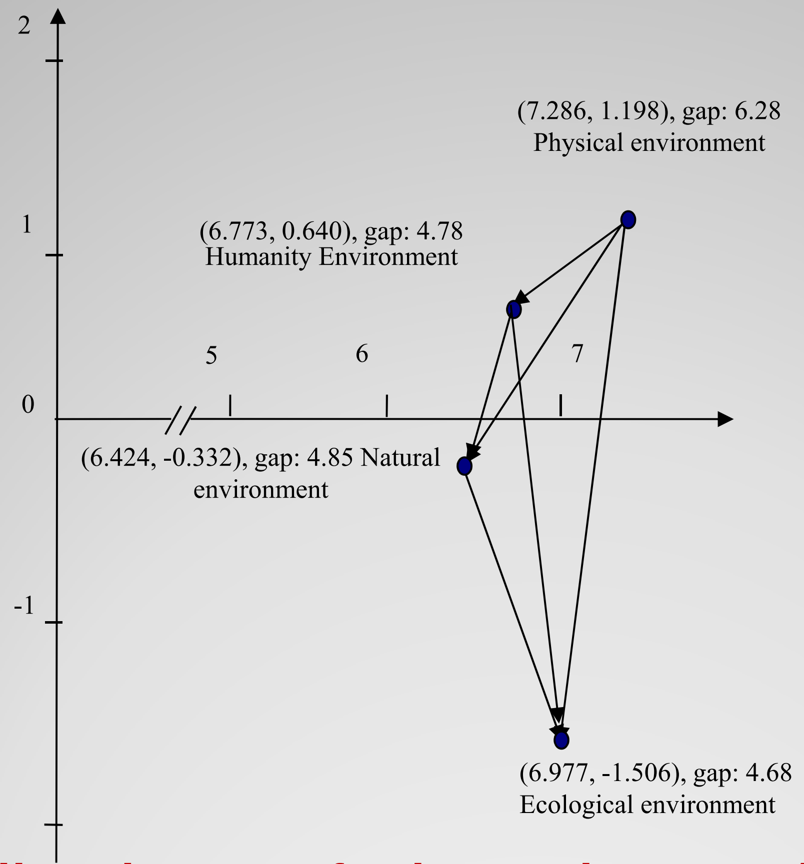
Definition 6

- Definition 6
  - Suppose r_i denotes the row sum of the i^{th} row of matrix $\boldsymbol{T}$. Then, r_i is the sum of the influences dispatching from factor i to the other all factors, both directly and indirectly. Suppose that d_j denotes the j^{th} column sum of the column of matrix $\boldsymbol{T}$. Then, d_j is the sum of the influences that factor j is received from the other all factors.

Definition 6 (Continued)

- Furthermore, when $i=j$ (i.e., the sum of the row sum and the column sum (r_i+d_j) represents the index representing the strength of the influence, both dispatching and received), (r_i+d_j) is the degree of the central role that factor i plays in the problem.
- If (r_i-d_j) is positive, then factor primarily is dispatching influence upon the other factors; and if (r_i-d_j) is negative, then factor primarily is received influence from other factors (Tamura et al., 2002; Tzeng et al., 2007; etc.).

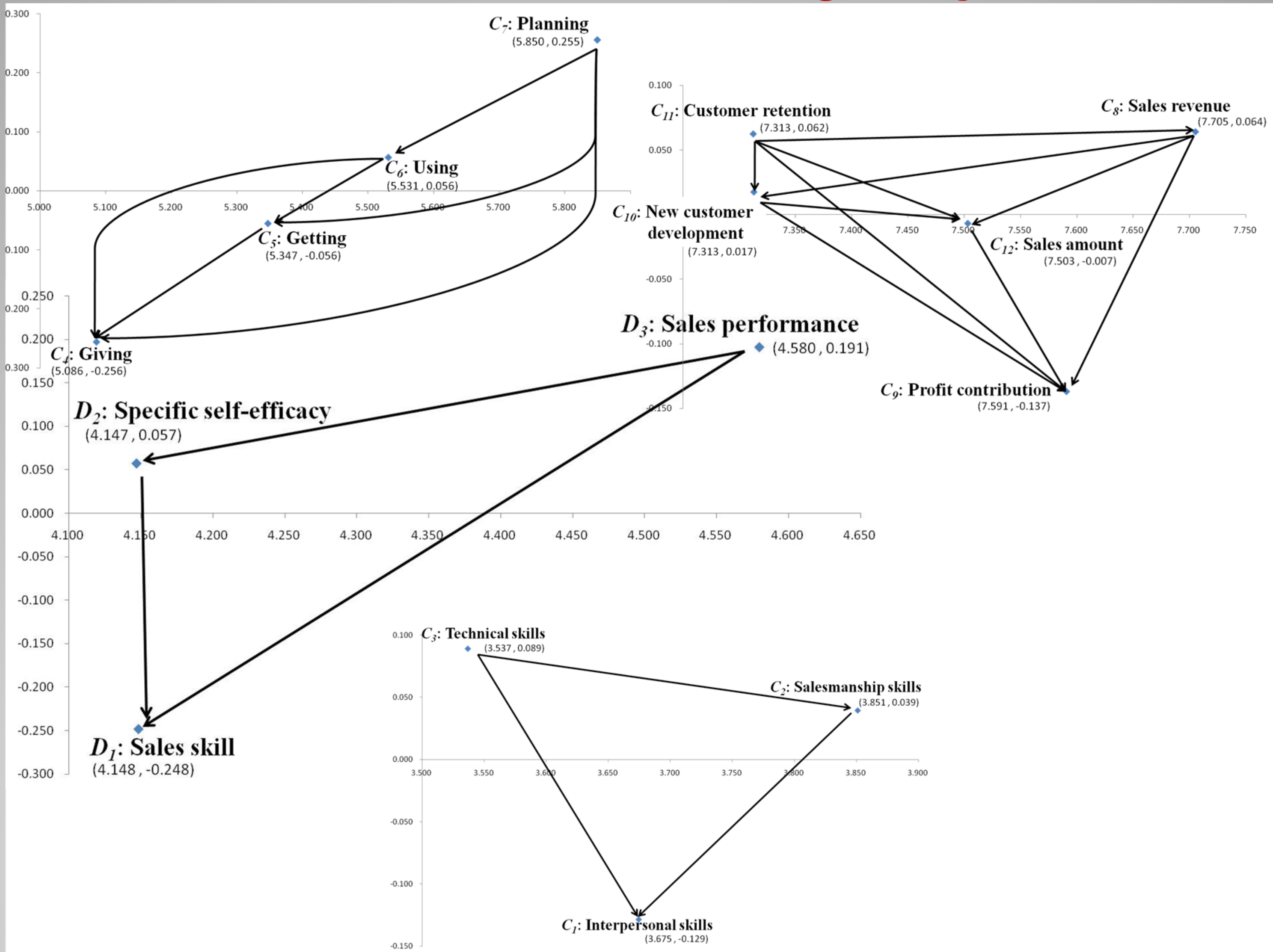
• **Example 1: For improving wetland environments**



The impact-direction map for improving gaps in performance values

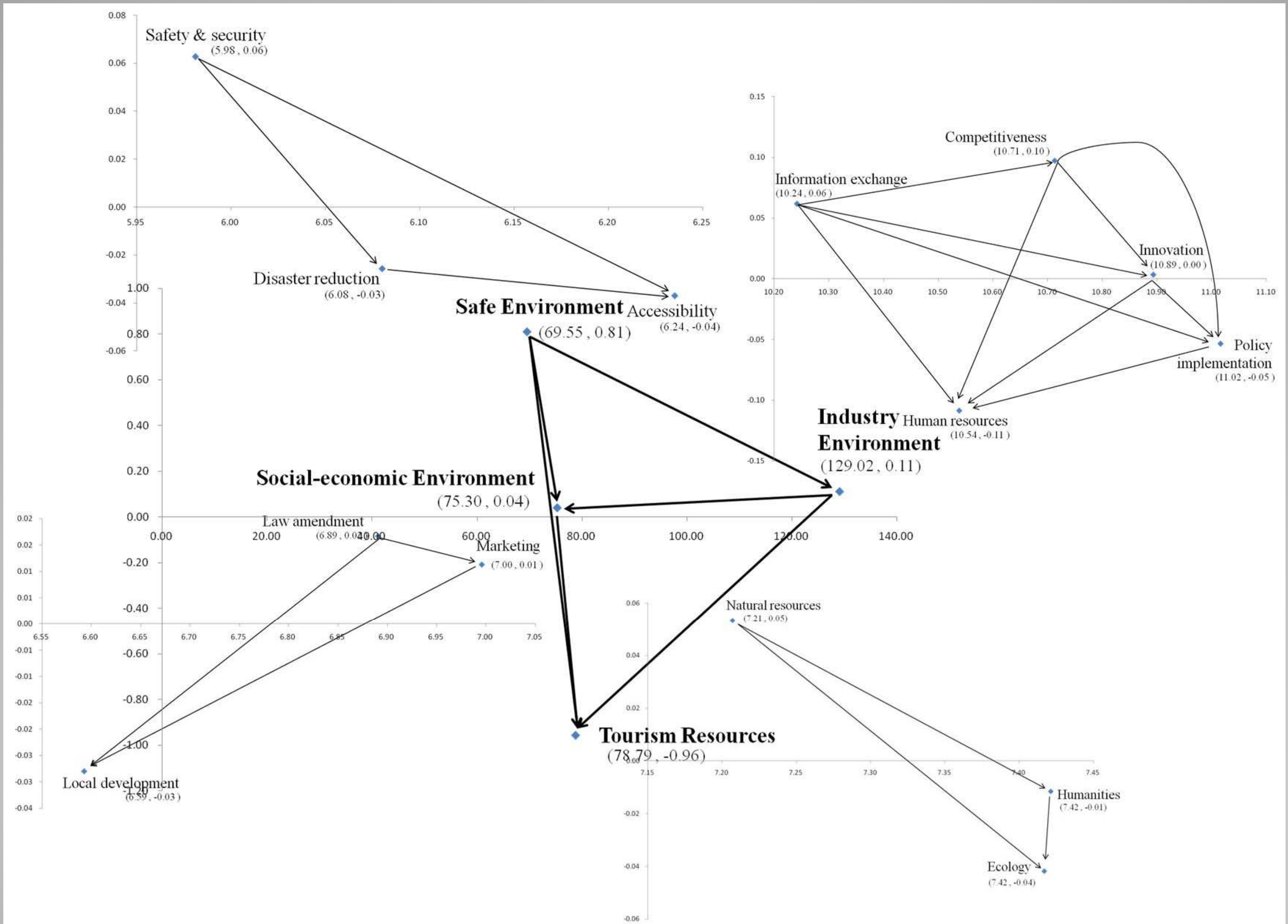
Chen, Y.C., Lien, H. P., Tzeng, G.H. (2010), Measures and evaluation for environment watershed plan using a novel hybrid MCDM model, *Expert Systems with Applications*, 37(2), *926-938*

Example 2: Strategies for improving cruise product sales in the travel agency



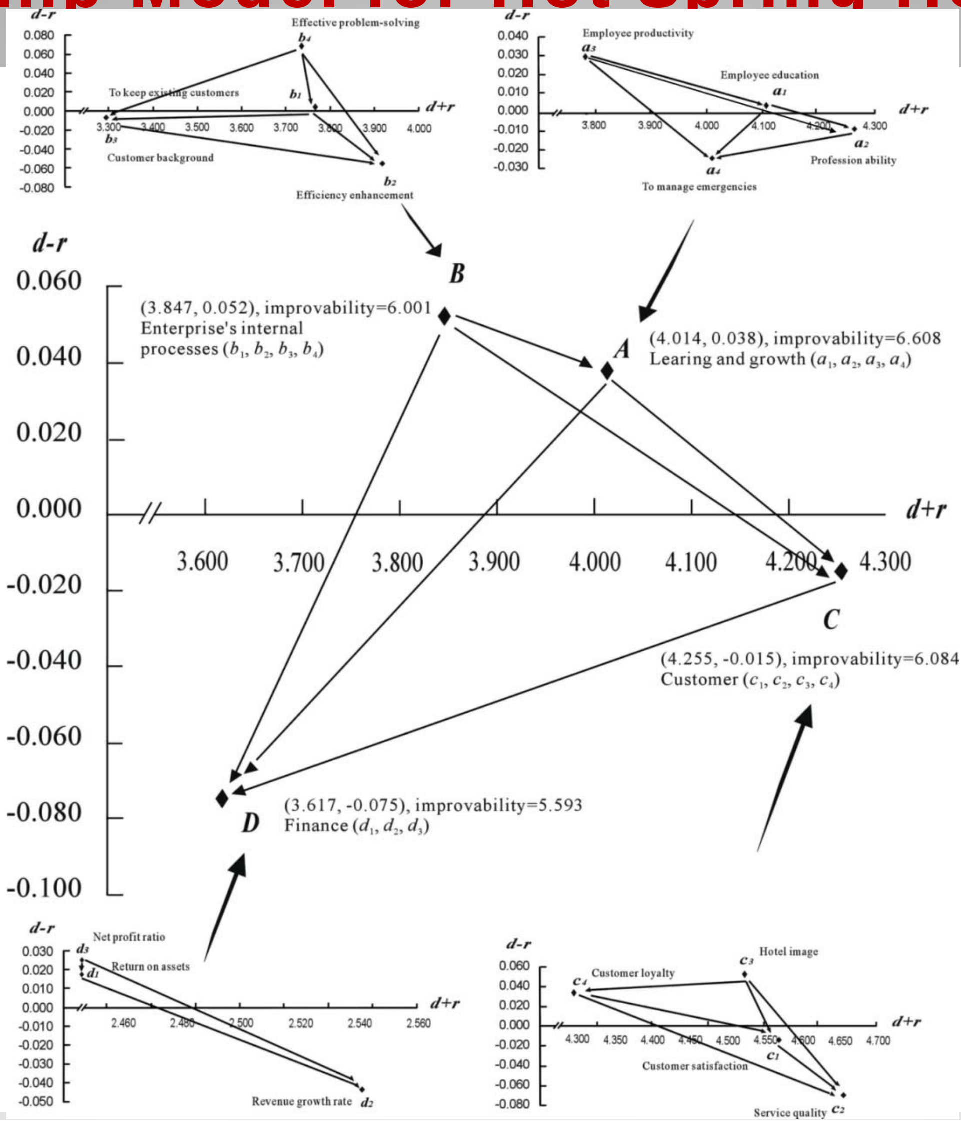
Liu, C. H., Tzeng, G.H., Lee, M.H. (2011), Strategies for improving cruise product sales in the travel agency- using hybrid MCDM models, The Service Industry Journal (Forthcoming).

Example 3: For improving tourism policy implementation



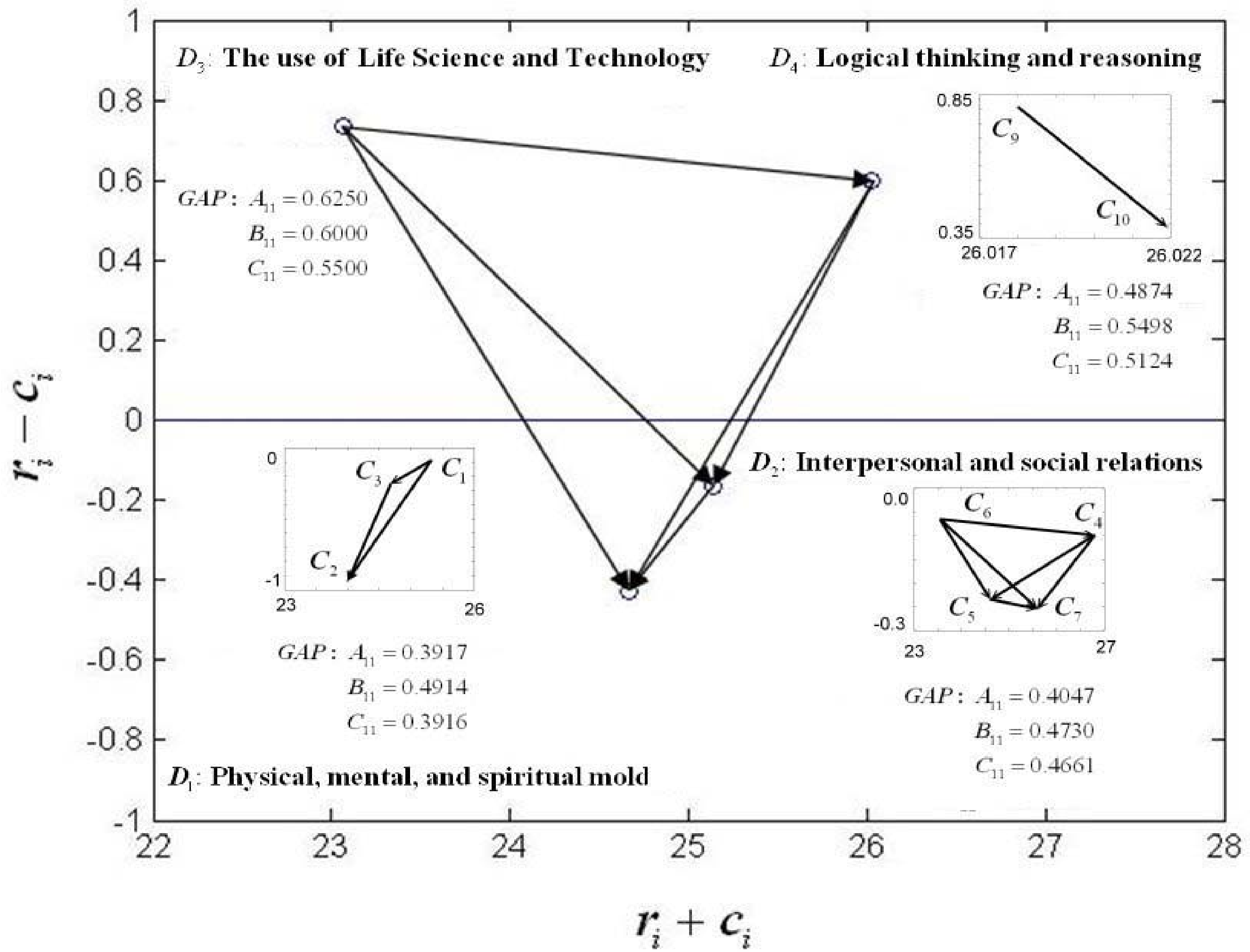
Liu, C.H., Tzeng, G.H., Lee, M.H. (2011), Improving tourism policy implementation - the use of hybrid MCDM models, Tourism Management (Accepted)

Example 4: Balanced Scorecard Approach to Establish a Performance Evaluation and Relationship Model for Hot Spring Hotels



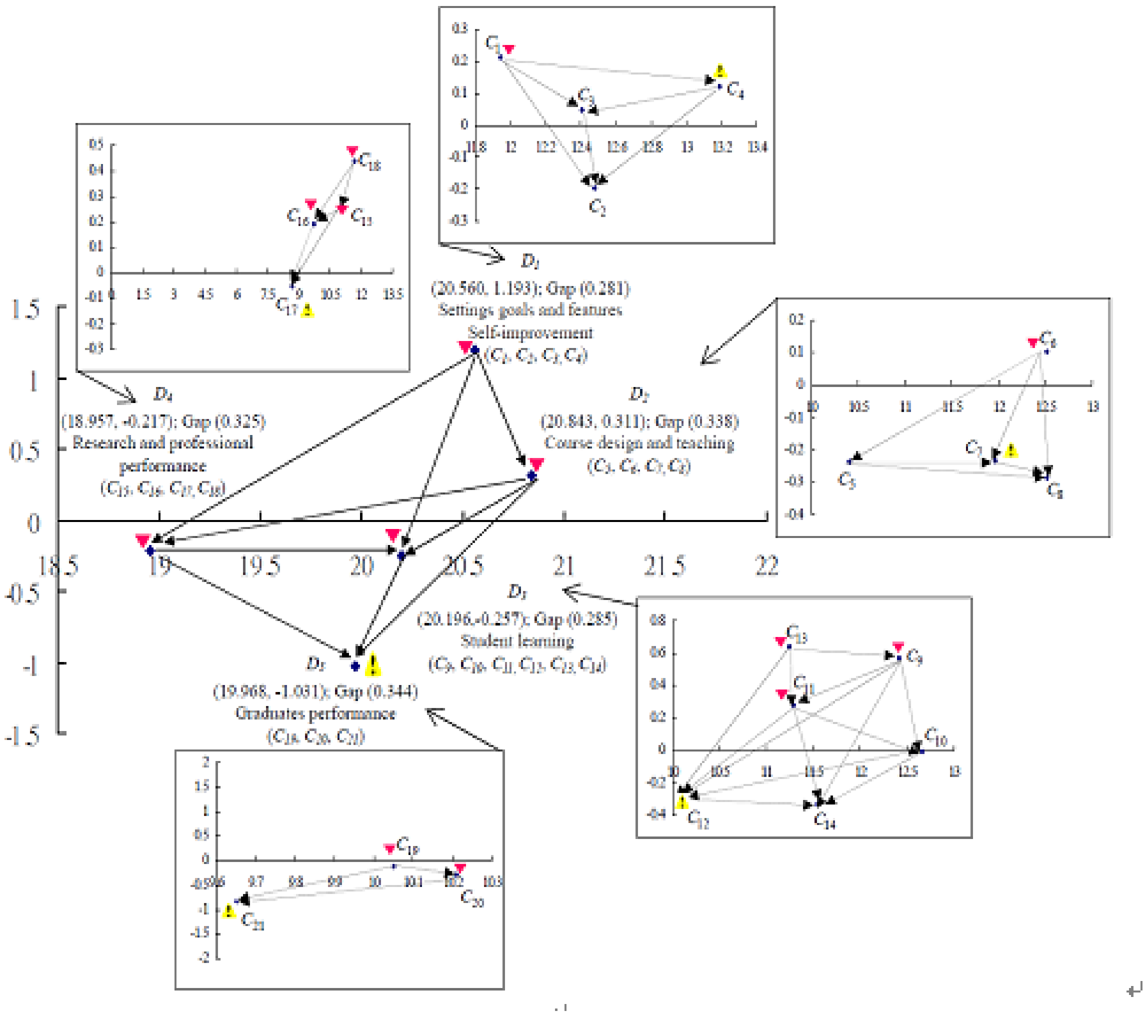
Chen, F.H., Hsu,T.S., Tzeng , G.H. (2011), A Balanced Scorecard Approach to Establish a Performance Evaluation and Relationship Model for Hot Spring Hotels Based on a Hybrid MCDM Model Combining DEMATEL and ANP, International Journal of Hospitality Management, 30(4), 908-932.

- **Example 5: Creating the Aspired Intelligent Assessment Systems for Teaching Materials: Case of Mandarin Chinese**



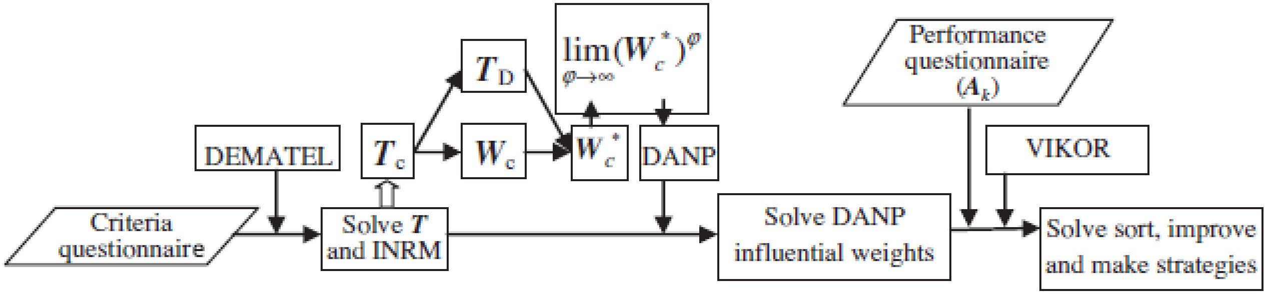
Chen, C.H. and Tzeng, G.H. (2011), Creating the Aspired Intelligent Assessment Systems for Teaching Materials, *Expert Systems with Applications*, 38(10), 12168-12179.

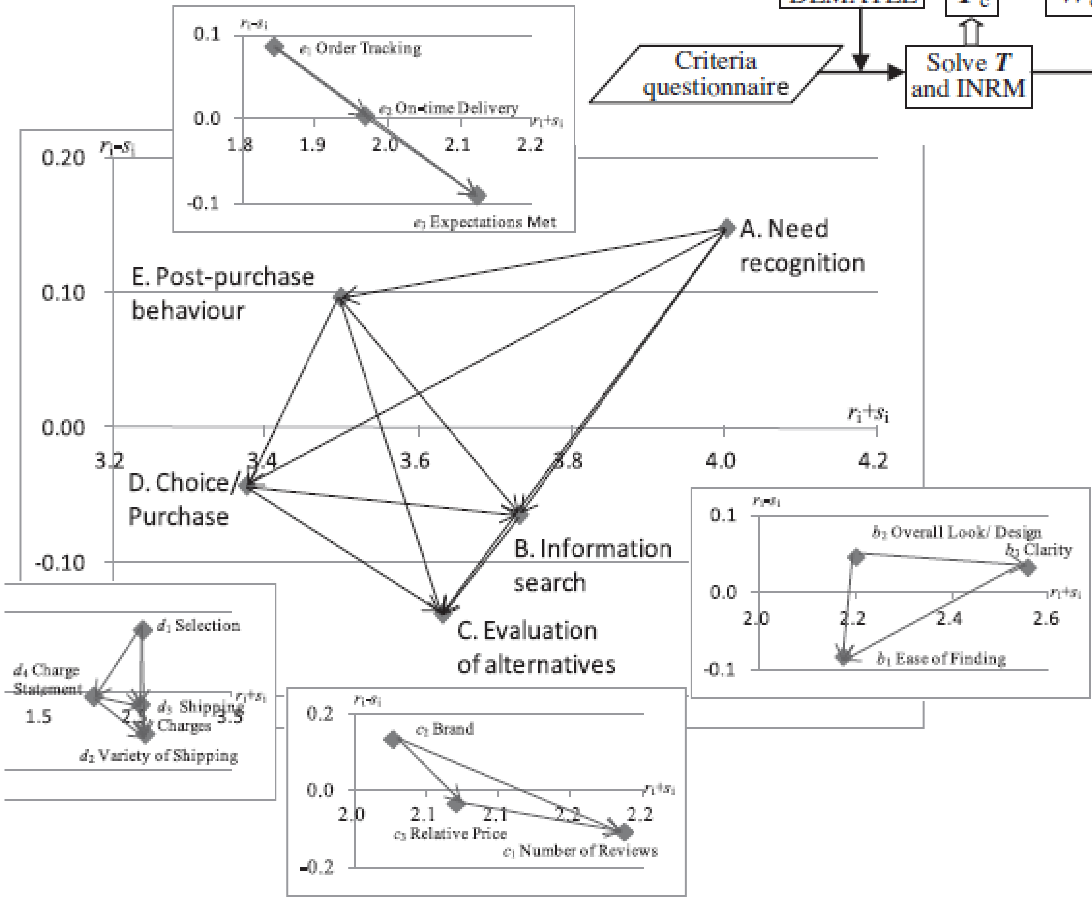
Example 6. For improve accreditation performance in higher education



Kua-Hsin Peng and **Gwo-Hshiung Tzeng**, "Strategies for Improving Accredition Performance in higher education institution, 4th International Conference on Computer Support Education (CSEDU 2012), Porto, Portugal, 16-18 April, 2012.

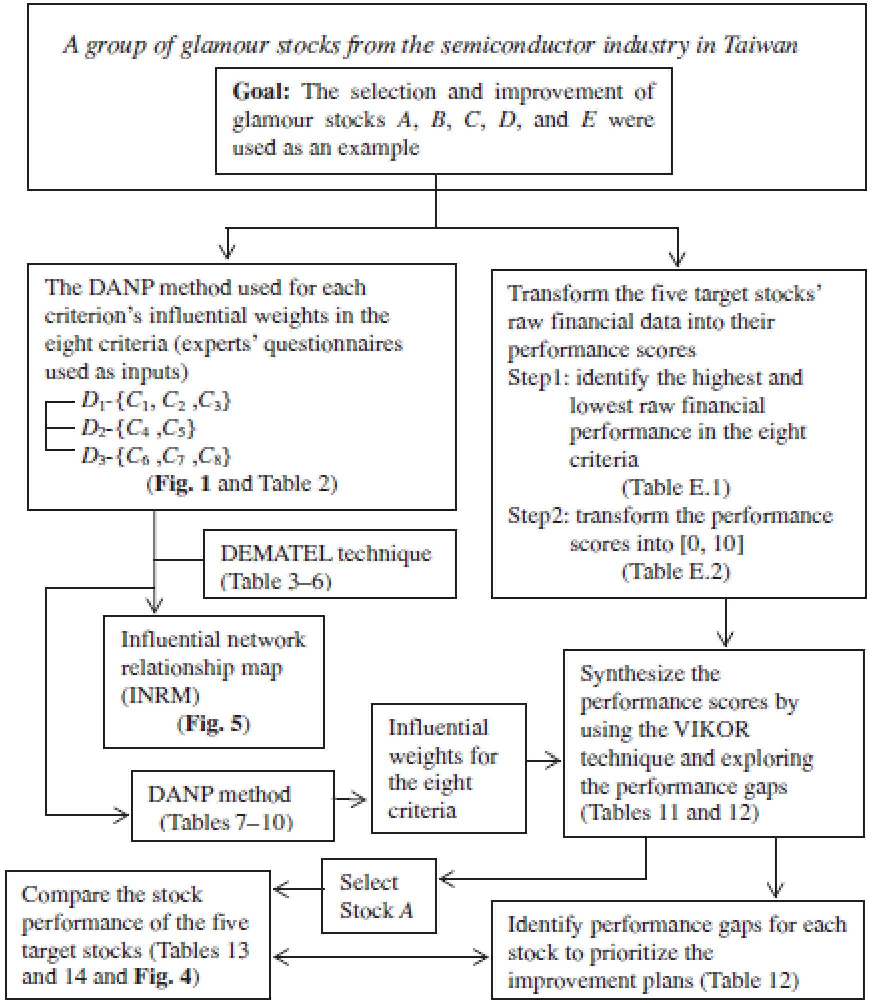
Example 7 Improve e-store business

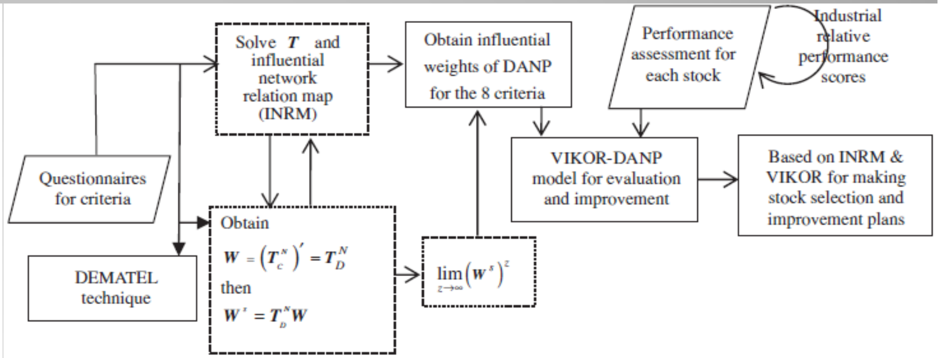




Wan-Yu Chiu, **Gwo-Hshiung Tzeng** (**Corresponding author**), Han-Lin Li (**2013**), **A new hybrid MCDM model combining DANP with VIKOR to improve e-store business**, ***Knowledge-Based Systems***, *Volume 37*, *January 2013*, *Pages 48-61* **(SCI**, **IF: 4.104**, 3.371 (5-years, 2012))

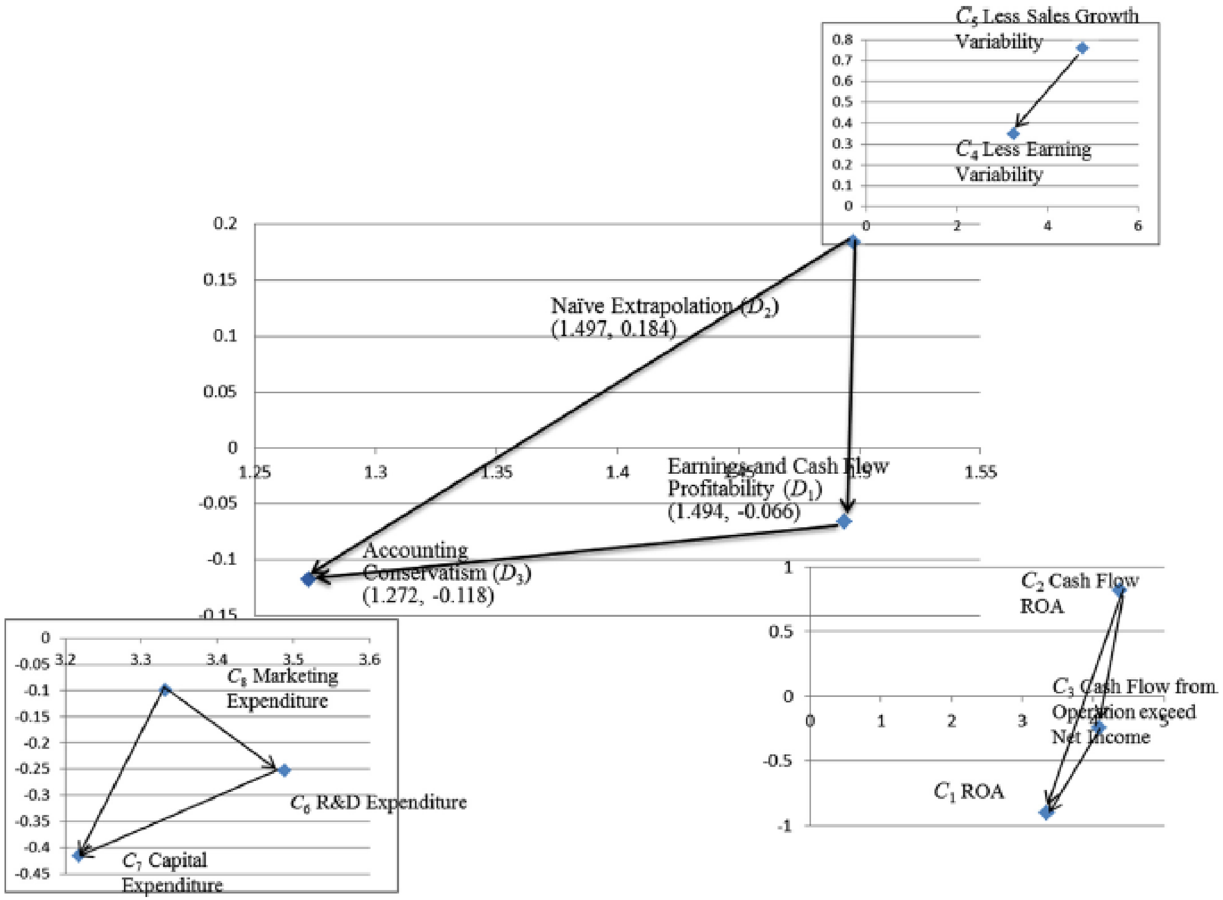
Example 8 Glamor stock selection and stock performance improvement



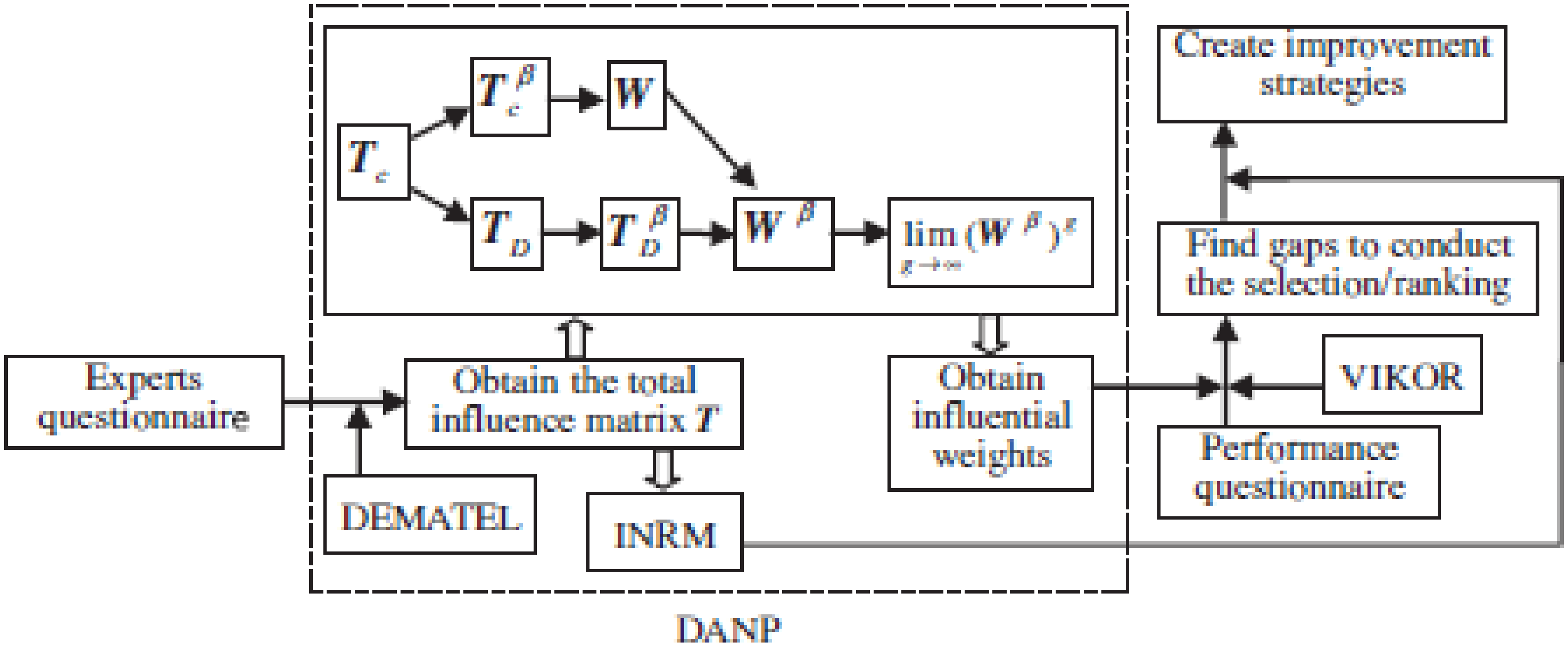


Kao-Yi Shen, Min-Ren Yan, and **Gwo-Hshiung Tzeng** (**Corresponding author**) **(2014)**, Combining VIKOR-DANP model for glamor stock selection and stock performance improvement, ***Knowledge-Based Systems***, *Volume 58, March 2014, Pages 86-97* **(SCI**, **IF: 4.104**, 3.371 (5-years, **2012**).

Example 8 Glamor stock selection and stock performance improvement

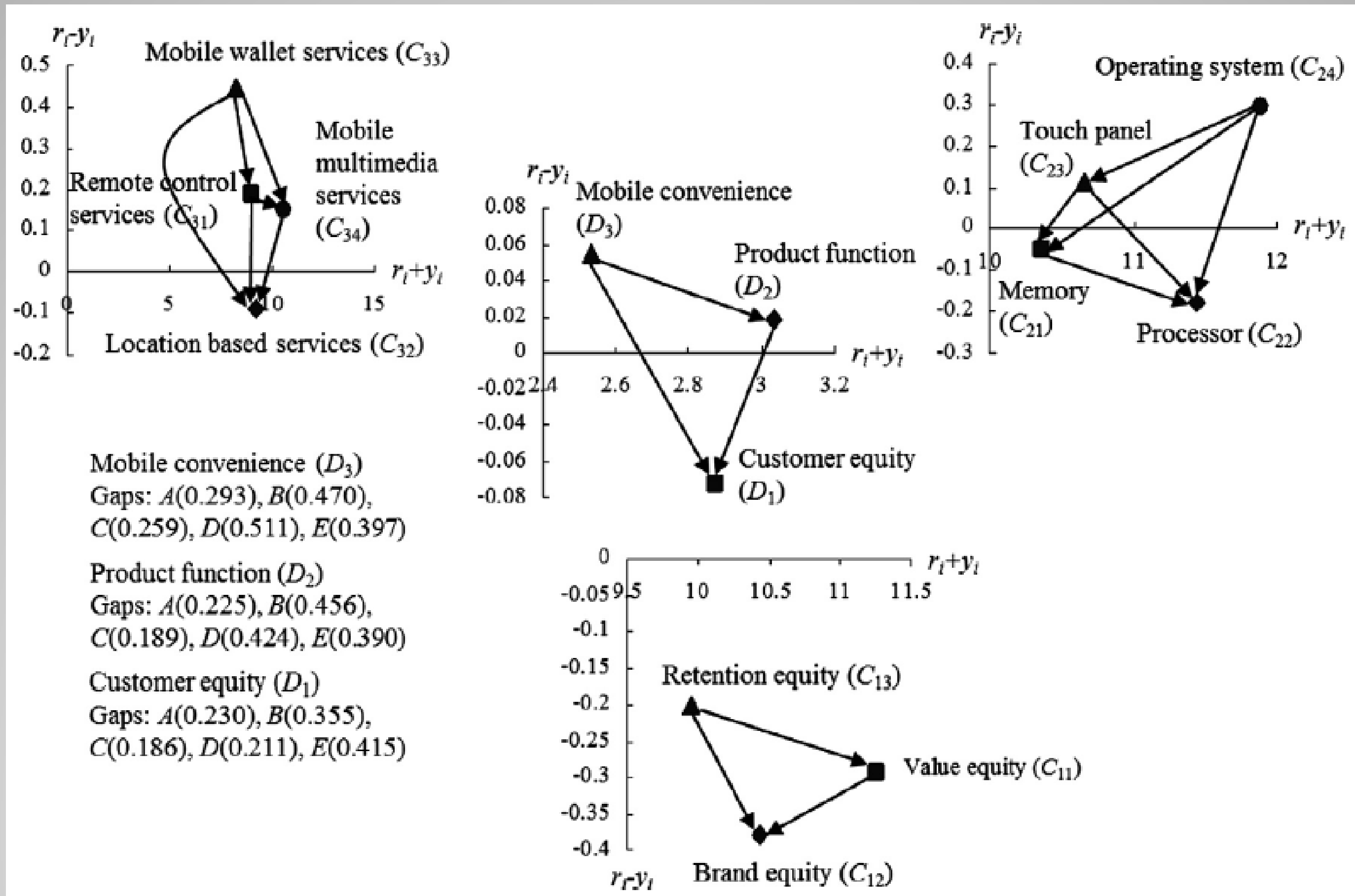


Example 9 Exploring smart phone improvements



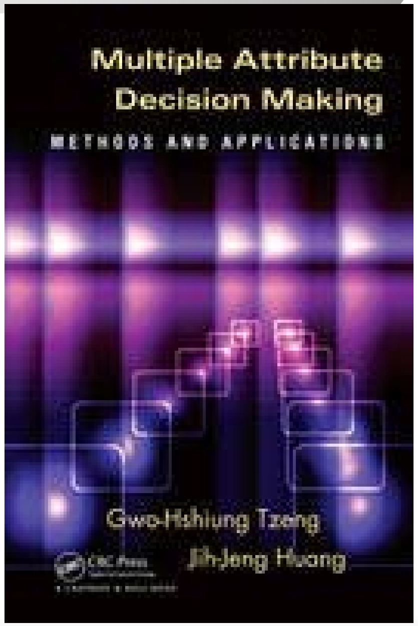
- Shu-Kung Hu, Ming-Tsang Lu, **Gwo-Hshiung Tzeng** (**Corresponding author**) **(2014)** Exploring smart phone improvements based on a hybrid MCDM model, **Expert Systems With Applications**, ***Volume 41, Issue 9***, ***July 2014***, ***Pages 4401-4413*** **(SCI**, **IF: 1.854**, 2.339 (5-years, **2012**).

Example 9 Exploring smart phone improvements



Analytic Network Process (ANP) and DANP (DEMATEL-based ANP)

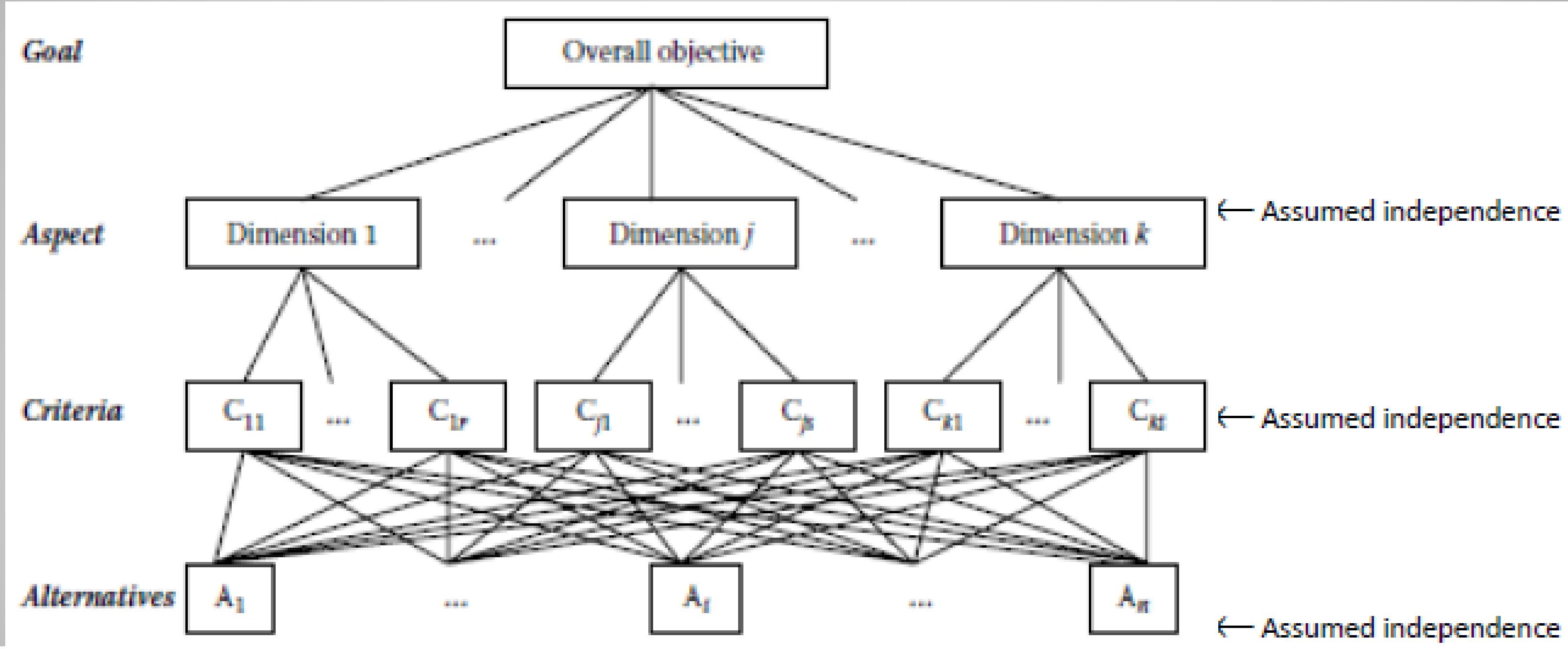
DANP (DEMATEL-based ANP) based on DEMATEL technique to build network relationship map (NRP) for constructing Super-matrix using the basic concept of ANP to find the influential weights (called DANP)



Source: Tzeng (2006)

Basic concept (1-1)

- **The AHP method**
  - A multi-criteria theory of measurement proposed by Saaty (1972, 1977, 1880, see my MADM book (2011)). Assumed independence in each aspect, criterion, and alternative.



Basic concept (1-2)

- **AHP and Fuzzy AHP**

Concepts of Pairwise Comparison for Solving AHP

$$
W \times w = \begin{matrix} \\ w_1 \\ \vdots \\ w_i \\ \vdots \\ w_n \end{matrix}
\overset{\begin{matrix} w_1 & \cdots & w_j & \cdots & w_n \end{matrix}}{\begin{bmatrix} w_1/w_1 & \cdots & w_1/w_j & \cdots & w_1/w_n \\ \vdots & & \vdots & & \vdots \\ w_i/w_1 & \cdots & w_i/w_j & \cdots & w_i/w_n \\ \vdots & & \vdots & & \vdots \\ w_n/w_1 & \cdots & w_n/w_j & \cdots & w_n/w_n \end{bmatrix}}
\begin{bmatrix} w_1 \\ \vdots \\ w_j \\ \vdots \\ w_n \end{bmatrix} = n \begin{bmatrix} w_1 \\ \vdots \\ w_i \\ \vdots \\ w_n \end{bmatrix}
$$

$$
Ww = nw \Rightarrow (W - nI)w = 0
$$

In real situations, w_i / w_j is unknown, but $a_{ij} \cong w_i / w_j$ and $a_{ij} = 1 / a_{ji}$ (positive reciprocal), and let $A = [a_{ij}]_{n \times n}$.

Basic concept (1-3)

- **AHP and Fuzzy AHP**

a. $Aw \cong nw \Rightarrow (A - \lambda_{\max} I)w = 0$, find $\lambda_{\max}$ and find w with $\lambda_{\max}$, and

calculate $C.I. = (\lambda_{\max} - n)/(n-1)$

$$\Rightarrow w = (w_1, w_2, \ldots, w_n)$$

b. $\min \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij} - \frac{w_i}{w_j})^2$

s.t. $\sum_{i=1}^{n} w_i = 1$; $w_i, w_j > 0$; $w_i, w_j \in \{1, 2, \ldots, n\}$

$$\Rightarrow w = (w_1, w_2, \ldots, w_n)$$

Basic concept (1-4)

- **AHP and Fuzzy AHP**

c. $r_i = (\prod_{j=1}^{n} a_{ij})^{1/n} \Rightarrow w_i = r_i / \sum_{i=1}^{n} r_i$ (normalization) $\Rightarrow \quad w = (w_1, w_2, \dots, w_n)$

d. When $Aw = \lambda_{\max} w$, then $\lambda_{\max}$ can be estimated by $\lambda_{\max} = \frac{1}{n} \sum_{i=1}^{n} \frac{(Aw)_i}{w_i}$

$\Rightarrow w = (w_1, w_2, \dots, w_n)$

- **Concepts of Pairwise Comparison for Solving Fuzzy AHP**

(1) **Fuzzy** $\tilde{A} = [\tilde{a}_{ij}]_{n \times n} \rightarrow$ **Fuzzy** $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)$

a. $\tilde{A} \rightarrow$ solve $\tilde{\lambda}_{\max} \rightarrow$ solve $\tilde{w}_i$, i.e. $(\tilde{A} - \tilde{\lambda}_{\max} I)\tilde{w} = 0 \Rightarrow \tilde{w} = (\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)$

Tzeng, G.H., Jen, W., Hu, K.C. (2002). Fuzzy factor analysis for selecting service quality factors-a case of the service quality of city bus service. International Journal of Fuzzy Systems, 4(4), 911-921.

Basic concept (1-4)

- **AHP and Fuzzy AHP**

(1) Fuzzy $\tilde{A} = [\tilde{a}_{ij}]_{n\times n} \rightarrow$ **Fuzzy** $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)$

b. $\tilde{r}_i = [\tilde{a}_{i1} \otimes \tilde{a}_{i2} \otimes \cdots \otimes \tilde{a}_{in}]^{1/n} \Rightarrow \tilde{w}_i = \tilde{r}_i \otimes [\tilde{r}_1 \oplus \tilde{r}_2 \oplus \cdots \oplus \tilde{r}_n]^{-1}$

Inverse operation of triangular fuzzy number: $(a,b,c)^{-1} = (1/c, 1/b, 1/a)$

$\Rightarrow \tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)$

(2) Fuzzy $\tilde{A} = [\tilde{a}_{ij}]_{n\times n} \rightarrow$ **Crisp** $w = (w_1, w_2, \ldots, w_n)$

c. $\tilde{A} = [\tilde{a}_{ij}]_{n\times n}$, $\tilde{a}_{ij} \cong \frac{w_i}{w_j}$, $l_{ij} \tilde{\leq} \frac{w_i}{w_j} \tilde{\leq} u_{ij}$, $i = 1, 2, \ldots, n-1; j = 1, 2, \ldots, n$; $i < j$

$l_{ij}(\alpha) \tilde{\leq} \frac{w_i}{w_j} \tilde{\leq} u_{ij}(\alpha)$ in level α, then fuzzy constraints:

$$\begin{aligned} w_i - w_j u_{ij}(\alpha) &\tilde{\leq} 0 \\ -w_i + w_j l_{ij}(\alpha) &\tilde{\leq} 0 \end{aligned} \Rightarrow \boldsymbol{Rw} \tilde{\leq} \mathbf{0}$$

where, the matrix $\boldsymbol{R} \in \Re^{m\times n}$, $m = n(n-1)$

Basic concept (1-4)

- **AHP and Fuzzy AHP**

then $\mu_k(R_k w) = \begin{cases} 1 - \dfrac{R_k w}{d_k}, & R_k w \le d_k \\ 0, & R_k w \le d_k \end{cases}$

$$\lambda = \mu_D(w) = \max_{w} \{ \min_{k=1,2,\dots,m} [\mu_1(R_1 w), \dots, \mu_m(R_m w)] \mid w \in Q^{n-1},\ w_1 + w_2 + \cdots + w_n = 1 \}$$

The max-min prioritization problem:

$$\max \ \lambda$$

s. t,

$$\lambda \le 1 - \frac{R_k w}{d_k}$$

$$\sum_{i=1}^{n} w_i = 1,\ w_i > 0,\ i = 1, 2, \dots, n;\ k = 1, 2, \dots, 2m$$

$$\Rightarrow w = (w_1, w_2, \dots, w_n)$$

Liou, J.J.H., Tzeng, G.H., Tsai, C.Y., CC Hsu, C.C. (3011). A hybrid ANP model in fuzzy environments for strategic alliance partner selection in the airline industry, Applied Soft Computing, 11(4), 3515–3524.

Mikhailov, L. (2003). Deriving priorities from fuzzy pairwise comparison judgments, Fuzzy Sets and System, 134(3), 365–385.

Basic concept (1-4)

- **AHP and Fuzzy AHP**

(3) **Crisp** $A=[a_{ij}]_{n\times n}$ $\rightarrow$ **Fuzzy** $\tilde{w}=(\tilde{w}_1,\tilde{w}_2,\ldots,\tilde{w}_n)$

$$w^k=(w_1^k,w_2^k,\ldots,w_n^k),\ k=1,2,\ldots,K;$$

$$\tilde{w}=(\tilde{w}_1,\ldots,\tilde{w}_j,\ldots,\tilde{w}_n);$$

$$\tilde{w}_j=(l_j,m_j,u_j);$$

where, $l_j=\min_k\{w_j^k \mid k=1,2,\ldots,K\};\ m_j=\frac{1}{K}\sum_{k=1}^{K}w_j^k$ or $m_j=[\prod_{k=1}^{K}w_j^k]^{1/K};$

$$u_j=\max_k\{w_j^k \mid k=1,2,\ldots,K\}.$$

Basic concept (1)

- **The ANP method**
  - A multi-criteria theory of measurement proposed by Saaty (1996).
  - Provides a general framework to deal with
    - Decisions without making assumptions about the independence of higher-level elements from lower level elements
    - About the independence of the elements within a level as in a hierarchy.

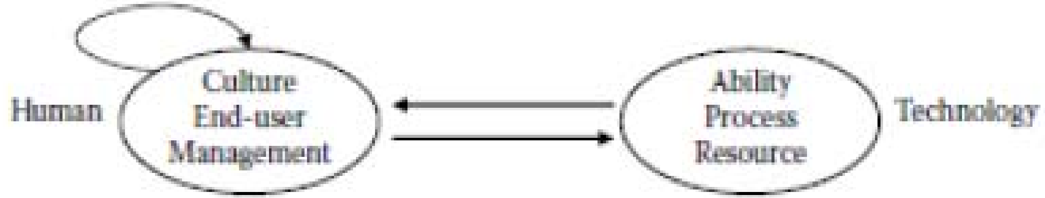
    [i.e., between each dimension is dependent and feedback, but criteria within dimension are also independent]

Basic concept (2)

- Compared with traditional MCDM methods, ANP is a more reasonable tool for dealing with complex MCDM problems in the real world.
  - Traditional MCDM methods usually assume the independence between criteria.
  - ANP extends AHP to deal with dependence and feedback problem and utilizes the super-matrix approach among clusters/dimensions, but criteria of each cluster/dimension are also assumed the independence as follows.

Basic concept (3)

- **ANP (Example)**



Self-relation

Independence in criteria (C, E, M)

Belong to “Human” dimension

	C	E	M	A	P	R
C	1	0	0	0.634	0.250	0.400
E	0	1	0	0.192	0.250	0.200
M	0	0	1	0.174	0.500	0.400
A	0.637	0.582	0.136	0	0	0
P	0.105	0.109	0.654	0	0	0
R	0.258	0.309	0.210	0	0	0

Independence in criteria (A, P, R)

Belong to “Technology” dimension

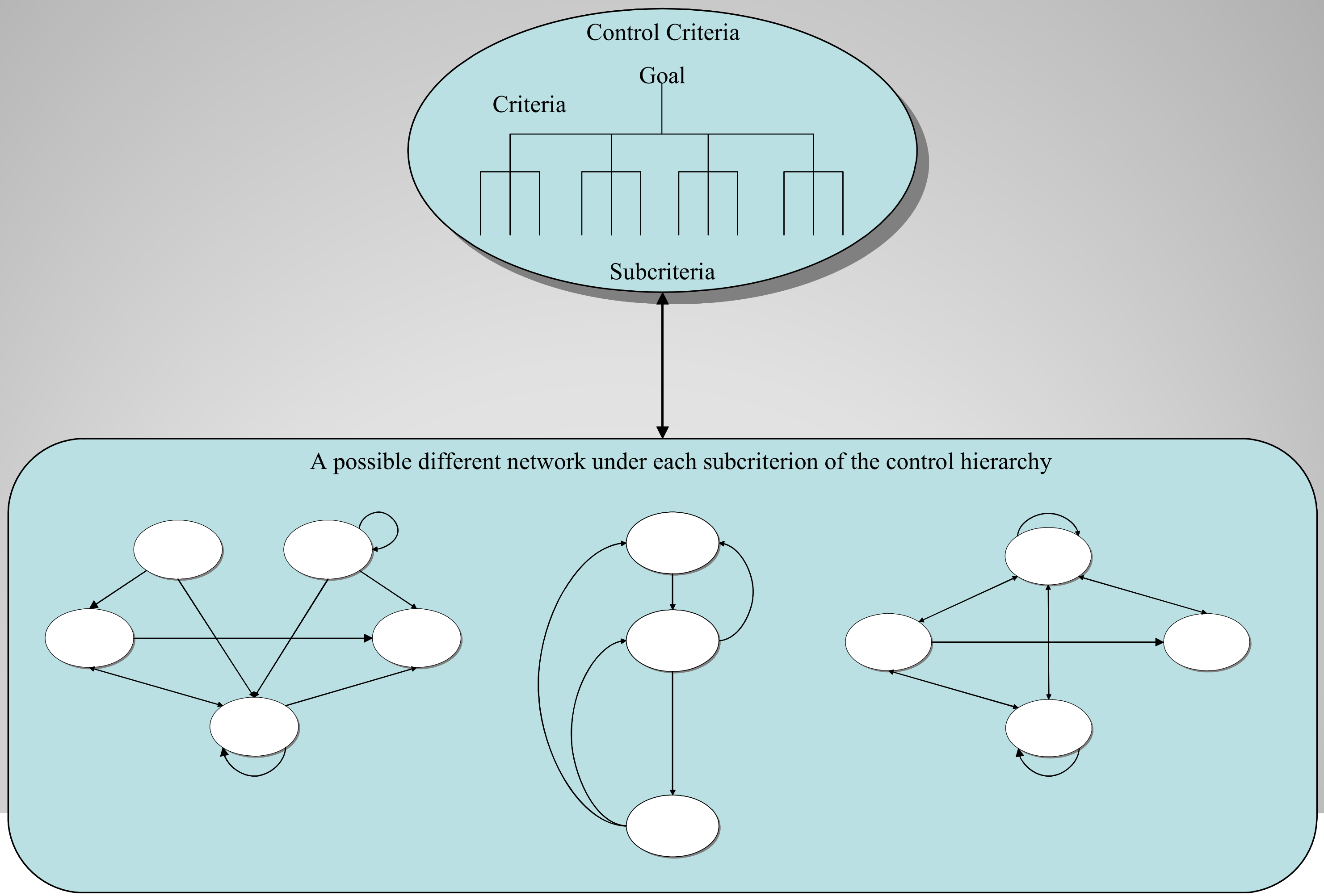
Basic concept (4)

- The ANP is a coupling of two parts.
  - The first consists of a control hierarchy or network of criteria and subcriteria that control the interactions.
  - The second is a network of influences among the elements and clusters.
    - The network varies from criterion to criterion
    - A different supermatrix of limiting influence is computed for each control criterion.
- Each of these super-matrices is weighted by the priority of its control criterion and the results are synthesized through addition for all the control criteria.

The Control Hierarchy (1)

- A control hierarchy is a hierarchy of criteria and subcriteria for which priorities are derived in the usual way with respect to the goal of the system being considered.
  - The criteria are used to compare the components of a system, and
  - The subcriteria are used to compare the elements.
  - The criteria with respect to which influence is presented in individual supermatrices are called control criteria.

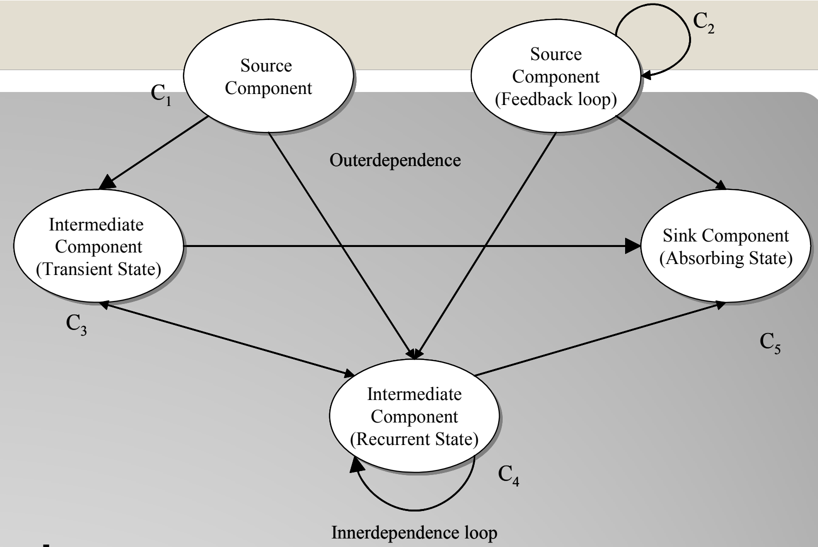
The Control Hierarchy (2)



The Network (1)

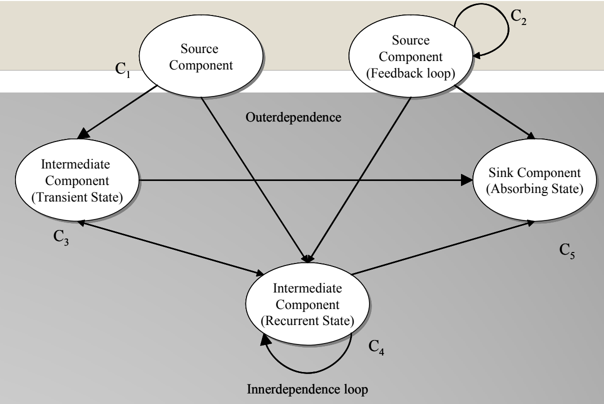
- A network connects the components of a decision system.
- According to size, there will be a system that is made up of subsystems, with each subsystem made up of components, and each component made up of elements.
- The elements in each component interact or have an influence on some or all of the elements of another component with respect to a property governing the interactions of the entire system, such as energy, capital, or political influence.

The Network (2)



- Source component
  - Those components which no arrow enters are known as source components. E.g. C_1 and C_2.
- Sink component
  - Those from which no arrow leaves are known as sink component. E.g. C_5.
- Transient component
  - Those components which arrows both enter and exit leave. E.g. C_3 and C_4.

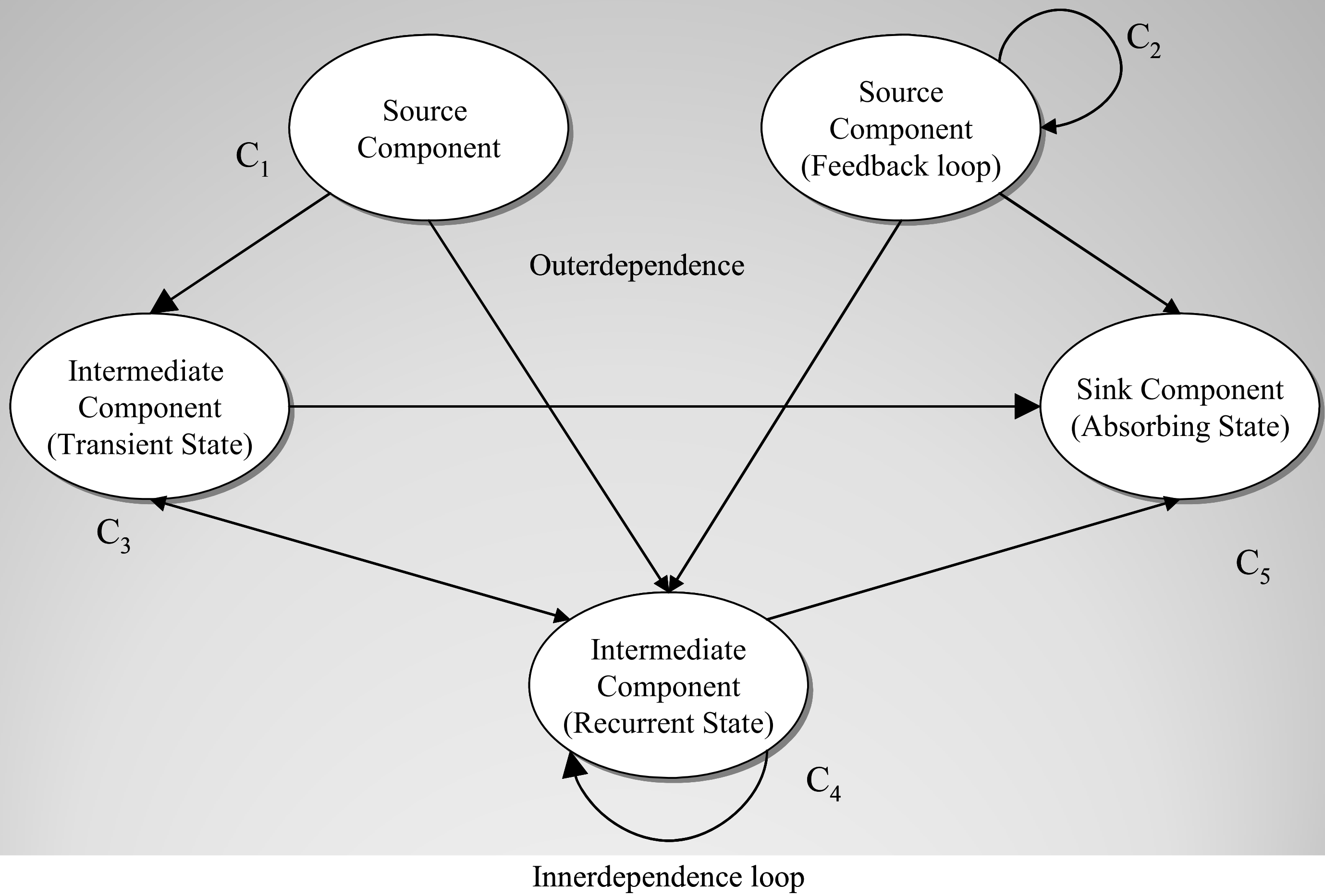
The Network (3)



- Cycle
  - A cycle of components is formed when the components feed back and forth into each other. E.g. C_3 and C_4.
- Loop
  - A loop connect to a component itself and is inner dependent. E.g.. C_2 and C_4 have loops that connect them to themselves and are inner dependent.
- Outer dependent
  - Other connections represent dependence between components which are thus known to be outer dependent.

The Network (4)

A Typical Example



The Super-matrix (1)

- A component of a decision network will be denoted by C_h, $h = 1,2,\ldots,m$, and assume that it has n_h elements, which we denote by e_{h1}, $e_{h2}, \ldots, e_{hm}$.
- The influences of a given set of elements in a component on any element in the decision system are represented by a ratio scale priority vector derived from pair-wise comparisons of the relative importance of one criterion and another criterion with respect to the interests or preferences of the decision-makers.

The Super-matrix (2)

- This relative importance value can be determined using a scale of 1 – 9 to represent equal importance to extreme importance.
- The influence of elements in the network on other elements in that network can be represented in the following supermatrix:

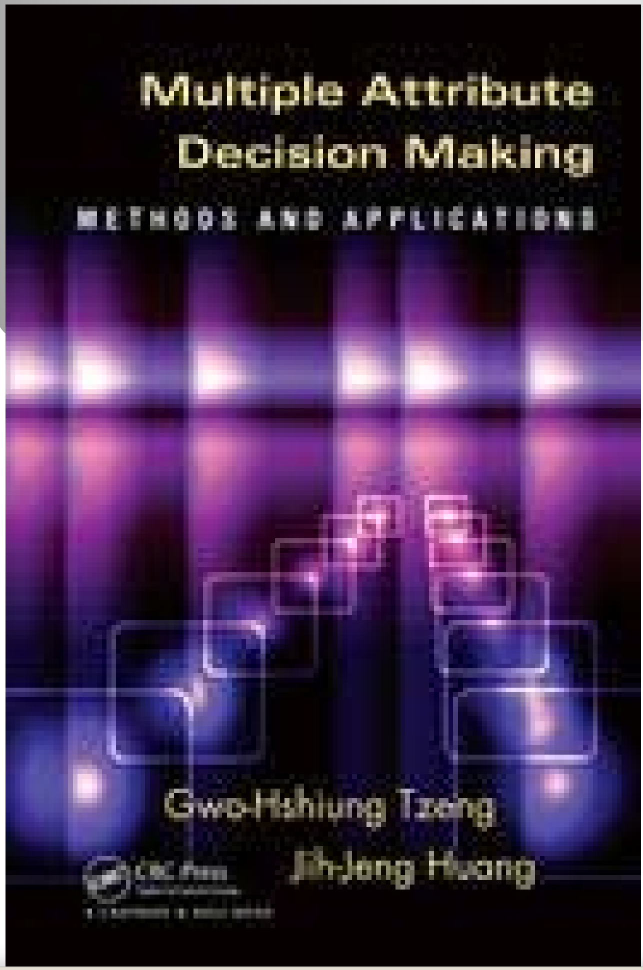
The Super-matrix (3)

- A typical entry W_{ij} in the supermatrix, is called a block of the super-matrix in the following form where each column of W_{ij} is a principal eigenvector of the influence of the elements in the ith component of the network on an element/ criterion in the jth component. Some of its entries may be zero corresponding to those elements/
- criteria that have no influence (are also assumed the independence of those elements/criteria).

The Supermatrix (4)

$$
W = \begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \\ e_{11} \cdots e_{1n_1} & e_{21} \cdots e_{2n_2} & \cdots & e_{m1} \cdots e_{mn_m} \end{array} \\
\begin{array}{cc} C_1 & \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1n_1} \end{array} \\ C_2 & \begin{array}{c} e_{21} \\ e_{22} \\ \vdots \\ e_{2n_2} \end{array} \\ \vdots & \vdots \\ C_m & \begin{array}{c} e_{m1} \\ e_{m2} \\ \vdots \\ e_{mn_m} \end{array} \end{array} &
\begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1m} \\ W_{21} & W_{22} & \cdots & W_{2m} \\ & & \ddots & \\ \vdots & \vdots & & \vdots \\ W_{m1} & W_{m2} & \cdots & W_{mm} \end{bmatrix}
\end{array}
$$

DEMATEL based Analytic Network Process (DANP)



New method
Hybrid MCDM model

- DEMATEL-based Analytic Network Process (DANP) (1/14)

➢The DANP is proposed by Pro. Tzeng, which is composed of DEMATEL technique and using the basic concept of ANP for determining the influential weights, called DANP (DEMATEL-based ANP).

DEMATEL-based ANP = DANP

- DEMATEL based Analytic Network Process (DANP) (2/14)

-

➢The DEMATEL technique was developed by the Battelle Geneva Institute:

(1) to analyze complex "real world problems" dealing mainly with interactive map-model techniques (Gabus & Fontela, 1972).

(2) to evaluate qualitative and factor-linked aspects of societal problems.

DEMATEL-based ANP = DANP

DEMATEL based Analytic Network Process (DANP) (3/14)

- The ANP method, a multi criteria theory of measurement developed by Saaty (Saaty, 1996) provides a general framework to deal with decisions without making assumptions about the independence of higher-level elements from lower level elements and about the independence of the elements within a level as in a hierarchy.

DEMATEL-based ANP = DANP

- DEMATEL based Analytic Network Process (DANP) (4/14) -

➢Step1: Calculate the direct-influence matrix by scores. Lead users and experts are asked to indicate the direct effect they believe a factor will have on factor , as indicated by . The matrix D of direct relations can be obtained.

➢Step2: Normalize the direct-influence matrix based on the direct-influence matrix D by the equation:

$$N = vD; v = \min\{1/\max_i \sum_{j=1}^{n} d_{ij}, 1/\max_j \sum_{i=1}^{n} d_{ij}\}, i, j \in \{1, 2, ..., n\}$$

DEMATEL-based ANP = DANP

- DEMATEL based Analytic Network Process (DANP) (5/14) -

- Step3: Attaining the total-influence matrix $\boldsymbol{T}$ by calculating this equation: $\boldsymbol{T} = \boldsymbol{N} + \boldsymbol{N}^2 + \ldots + \boldsymbol{N}^h = \boldsymbol{N}(\boldsymbol{I} - \boldsymbol{N})^{-1}, \quad \text{when } h \rightarrow \infty$

- Step4: The row and column sums are separately denoted as and within the total-relation matrix through equations:

$$\boldsymbol{T} = [t_{ij}], \quad i, j \in \{1, 2, \ldots, n\}$$

$$\boldsymbol{r} = [r_i]_{n\times 1} = \left[\sum_{j=1}^{n} t_{ij}\right]_{n\times 1} \qquad \boldsymbol{c} = [c_j]_{1\times n} = \left[\sum_{i=1}^{n} t_{ij}\right]_{1\times n}$$

DEMATEL-based ANP = DANP

DEMATEL based Analytic Network Process (DANP) (6/14)

Total relationship matrix T can be measured by criteria, shown as T_c

$$T_C = \begin{array}{c} \\ D_1 \\ \\ \vdots \\ \\ D_i \\ \\ \vdots \\ \\ D_m \end{array} \begin{array}{c} \\ c_{11} \\ c_{12} \\ \vdots \\ c_{1m_1} \\ \vdots \\ c_{i1} \\ c_{i2} \\ \vdots \\ c_{im_i} \\ \vdots \\ c_{m1} \\ c_{m2} \\ \vdots \\ c_{mm_m} \end{array} \overset{\begin{array}{ccccc} D_1 & & D_j & & D_m \\ c_{11}\ldots c_{1m_1} & \cdots & c_{j1}\ldots c_{jm_j} & \cdots & c_{m1}\ldots c_{mm_m} \end{array}}{\begin{bmatrix} T_c^{11} & \cdots & T_c^{1j} & \cdots & T_c^{1m} \\ \vdots & & \vdots & & \vdots \\ T_c^{i1} & \cdots & T_c^{ij} & \cdots & T_c^{im} \\ \vdots & & \vdots & & \vdots \\ T_c^{m1} & \cdots & T_c^{mj} & \cdots & T_c^{mm} \end{bmatrix}}_{n\times n|m<n,\ \sum_{j=1}^{m} m_j = n}$$

DEMATEL based Analytic Network Process (DANP) (7/14)

Step 5: Normalize $\boldsymbol{T}_c$ with the total degree of effect and obtain $\boldsymbol{T}_C^{\alpha}$

$$\boldsymbol{T}_C^{\alpha} = \begin{array}{c} \\ D_1 \\ \vdots \\ D_i \\ \vdots \\ D_m \end{array} \begin{array}{c} \\ c_{11} \\ c_{12} \\ \vdots \\ c_{1m_1} \\ \vdots \\ c_{i1} \\ c_{i2} \\ \vdots \\ c_{im_i} \\ \vdots \\ c_{m1} \\ c_{m2} \\ \vdots \\ c_{mm_m} \end{array} \overset{\begin{array}{ccccc} D_1 & & D_j & & D_m \\ c_{11}\ldots c_{1m_1} & \cdots & c_{j1}\ldots c_{jm_j} & \cdots & c_{n1}\ldots c_{mm_m} \end{array}}{\begin{bmatrix} \boldsymbol{T}_c^{\alpha 11} & \cdots & \boldsymbol{T}_c^{\alpha 1j} & \cdots & \boldsymbol{T}_c^{\alpha 1m} \\ \vdots & & \vdots & & \vdots \\ \boldsymbol{T}_c^{\alpha i1} & \cdots & \boldsymbol{T}_c^{\alpha ij} & \cdots & \boldsymbol{T}_c^{\alpha im} \\ \vdots & & \vdots & & \vdots \\ \boldsymbol{T}_c^{\alpha m1} & \cdots & \boldsymbol{T}_c^{\alpha mj} & \cdots & \boldsymbol{T}_c^{\alpha mm} \end{bmatrix}}_{n \times n | m < n, \sum_{j=1}^{m} m_j = n}$$

- DEMATEL based Analytic Network Process (DANP) (8/14) -

- According to the result of step 4
  - $(r_i + c_i)$ represents the index representing the strength of the influence, both dispatching and receiving, it is the degree of the central role that factor plays in the problem.
  - If $(r_i - c_i)$ is positive, then factor primarily is dispatching influence upon the strength of other factors; and if $(r_i - c_i)$ is negative, then factor primarily is receiving influence from other factors (Huang et al.,2007; Liou et al., 2007; Tamura et al., 2002).

DEMATEL-based ANP = DANP

- DEMATEL based Analytic Network Process (DANP) (9/14) -

➢Now we call the total-influence matrix $\boldsymbol{T}_C = \left[t_{ij}\right]_{nxn}$ obtained by criteria and $\boldsymbol{T}_D = \left[t_{ij}^D\right]_{nxn}$ obtained by dimensions (clusters) from $\boldsymbol{T}_C$.

➢ Then we normalize the unweighted supermatrix $\boldsymbol{W}$ based on weights of dimensions (clusters) by using the normalized influence matrix $\boldsymbol{T}_D$.

$$\boldsymbol{T}_D = \begin{bmatrix} t_{11}^{D_{11}} & \cdots & t_{1j}^{D_{1j}} & \cdots & t_{1m}^{D_{1m}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ t_{i1}^{D_{i1}} & \cdots & t_{ij}^{D_{ij}} & \cdots & t_{im}^{D_{im}} \\ \vdots & \vdots & & \vdots & \vdots \\ t_{m1}^{D_{m1}} & \cdots & t_{mj}^{D_{mj}} & \cdots & t_{mm}^{D_{mm}} \end{bmatrix}$$

$$\rightarrow d_1 = \sum_{j=1}^{m} t_{1j}^{D_{1j}}$$

$$\rightarrow d_i = \sum_{j=1}^{m} t_{ij}^{D_{ij}} \quad , d_i = \sum_{j=1}^{m} t_{ij}^{D_{ij}}, i = 1,\ldots,m$$

$$\rightarrow d_m = \sum_{j=1}^{m} t_{mj}^{D_{mj}}$$

DEMATEL-based ANP = DANP

- DEMATEL based Analytic Network Process (DANP) (11/14)
-

➢ Step 6: normalize the total-influence matrix and represent it as T_D

$$T_D^{\alpha} = \begin{bmatrix} t_{11}^{D_{11}}/d_1 & \cdots & t_{1j}^{D_{1j}}/d_1 & \cdots & t_{1m}^{D_{1m}}/d_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ t_{i1}^{D_{i1}}/d_i & \cdots & t_{ij}^{D_{ij}}/d_i & \cdots & t_{im}^{D_{im}}/d_i \\ \vdots & \vdots & & \vdots & \vdots \\ t_{m1}^{D_{m1}}/d_m & \cdots & t_{mj}^{D_{mj}}/d_m & \cdots & t_{mm}^{D_{mm}}/d_m \end{bmatrix} = \begin{bmatrix} t_D^{\alpha 11} & \cdots & t_D^{\alpha 1j} & \cdots & t_D^{\alpha 1m} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha i1} & \cdots & t_D^{\alpha ij} & \cdots & t_D^{\alpha im} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha m1} & \cdots & t_D^{\alpha mj} & \cdots & t_D^{\alpha mm} \end{bmatrix}$$

DEMATEL-based ANP = DANP

DEMATEL based Analytic Network Process (DANP) (12/14)

➢Step 7: Calculate the un-weighted super-matrix $\boldsymbol{W}$ based on $\boldsymbol{T}_c^{\alpha}$.

$$\boldsymbol{W} = (\boldsymbol{T}_c^{\alpha})' = \begin{array}{cc} & \begin{array}{ccccc} D_1 & & D_i & & D_m \\ c_{11}\ldots c_{1m_1} & \cdots & c_{i1}\ldots c_{im_i} & \cdots & c_{m1}\ldots c_{mm_m} \end{array} \\ \begin{array}{cc} D_1 & \begin{array}{c} c_{11} \\ c_{12} \\ \vdots \\ c_{1m_1} \end{array} \\ \vdots & \vdots \\ D_j & \begin{array}{c} c_{j1} \\ c_{j2} \\ \vdots \\ c_{jm_j} \end{array} \\ \vdots & \vdots \\ D_m & \begin{array}{c} c_{m1} \\ c_{m2} \\ \vdots \\ c_{mm_m} \end{array} \end{array} & \begin{bmatrix} \boldsymbol{W}^{11} & \cdots & \boldsymbol{W}^{i1} & \cdots & \boldsymbol{W}^{m1} \\ \vdots & & \vdots & & \vdots \\ \boldsymbol{W}^{1j} & \cdots & \boldsymbol{W}^{ij} & \cdots & \boldsymbol{W}^{mj} \\ \vdots & & \vdots & & \vdots \\ \boldsymbol{W}^{1m} & \cdots & \boldsymbol{W}^{im} & \cdots & \boldsymbol{W}^{mm} \end{bmatrix}_{n \times n | m < n, \sum_{j=1}^{m} m_j = n} \end{array}$$

DEMATEL-based ANP = DANP

DEMATEL based Analytic Network Process (DANP) (13/14)

➢Step 8: Calculate the weighted supermatrix W^{α}.

$$W^{\alpha} = T_D^{\alpha} W = \begin{bmatrix} t_D^{\alpha 11} \times W^{11} & \cdots & t_D^{\alpha i1} \times W^{i1} & \cdots & t_D^{\alpha m1} \times W^{m1} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha 1j} \times W^{1j} & \cdots & t_D^{\alpha ij} \times W^{ij} & \cdots & t_D^{\alpha mj} \times W^{mj} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha 1m} \times W^{1m} & \cdots & t_D^{\alpha im} \times W^{im} & \cdots & t_D^{\alpha mm} \times W^{mm} \end{bmatrix}_{n \times n | m < n, \sum_{j=1}^{m} m_j = n}$$

DEMATEL-based ANP = DANP

DEMATEL based Analytic Network Process (DANP) (14/14)

➢Step 9: Limit the weighted super-matrix by raising it to a sufficiently large power z, as this equation, until the super-matrix has converged and become a long-term stable super-matrix to get the global priority influential vectors or called DANP influential weights.

$$\lim_{z\to\infty}(\boldsymbol{W}^{\alpha})^{z}$$

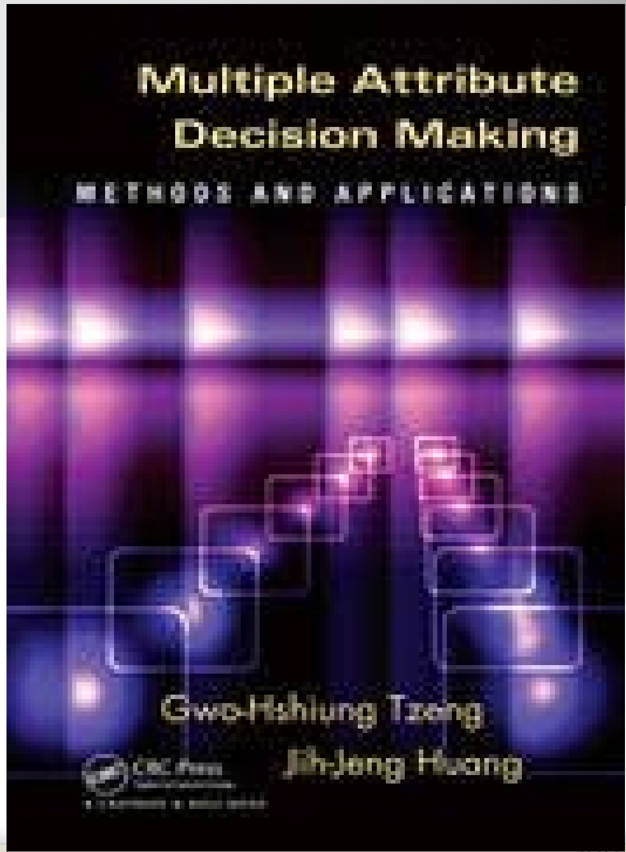
Then the vector of influential weights $w = (w_1, \ldots, w_j, \ldots, w_n)$ can be obtained.

DEMATEL-based ANP = DANP

The Modified VIKOR method –

Using "**aspired-worst**" **as benchmark** to **replace** traditional "traditional "**max-min**" **as benchmark** for **normalized the performance gap matrix** in each criterion, then **minimize average gaps** for all dimensions/criteria and **improve the maximal gaps for priority improvement** as strategies based on influential network relation map (INRM).

New MADM Methods →



The Modified VIKOR method (1)

- The rating performance scores are normalised by the best value and the worst value; for example, the scale **performance scores from 0 (the worst value**, $f_j^{worst}=0$) **to 10 (the best value**, called **the aspiration level**, $f_j^{aspired}=10$), and the scores of the criterion are denoted by f_{ij} **for an alternative as gap**. **The new modified VIKOR is more appropriate to the analysis of real-world situations. T**hese models can be used to resolve other real business questions.

The Modified VIKOR method (2)

Development of the modified VIKOR method began with the following form of L_p-metric:

$$L_k^p = \left\{ \sum_{j=1}^{n} [w_j (| f_j^{aspired} - f_{kj} |) / (| f_j^{aspired} - f_j^{worst} |]^p \right\}^{1/p}$$

where $1 \leq p \leq \infty; k = 1,2,\ldots,K$ and influential weight w_j is derived from the DANP. To formulate the ranking and gap measure $L_k^{p=1}$ (as S_k) and $L_k^{p=\infty}$ (as Q_k) are used by modified VIKOR method (Tzeng et al., 2002, 2005; Opricovic and Tzeng, 2002, 2004, 2007).

$$S_k = L_k^{p=1} = \sum_{j=1}^{n} w_j r_{kj} = \sum_{j=1}^{n} w_j (| f_j^{aspired} - f_{kj} |) / (| f_j^{aspired} - f_j^{worst} |)$$

$$Q_k = L_k^{p=\infty} = \max_j \{(| f_j^{aspired} - f_{kj} |) / (| f_j^{aspired} - f_j^{worst} |) \mid j = 1,2,\ldots,n\}$$

VIKOR method (3)

- The new VIKOR method consists of the following:
- **Step 1: Finding the normalised gap.**

$$r_{kj} = \left(\left| f_j^{aspired} - f_{kj} \right|\right) \Big/ \left(\left| f_j^{aspired} - f_j^{worst} \right|\right)$$

- **Step 2: Computing the gap for minimal and the maximal gap for priority improvement.**

$$S_k = L_k^{p=1} = \sum_{j=1}^{n} w_j \times r_{kj}, \qquad \forall k$$

$$Q_k = L_k^{p=\infty} = \max_j \{ r_{kj} | j = 1,2,\ldots,n \}, \qquad \forall k$$

VIKOR method (4)

Step 3: Obtaining the comprehensive indicator

Based on the above concepts, the comprehensive indicator of the compromise VIKOR can be written as follow.

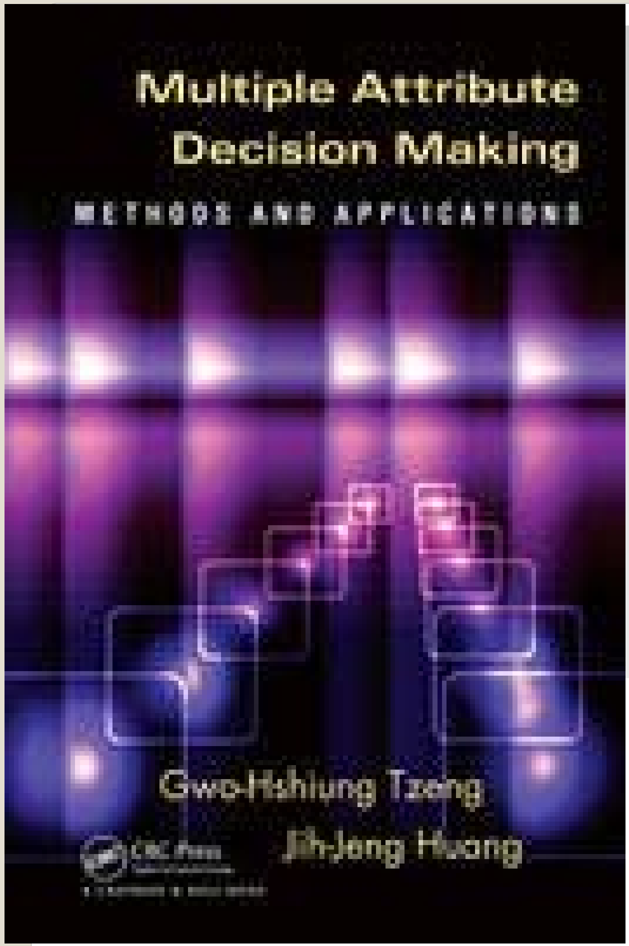
$$R_k = v(S_k - S^*)/(S^- - S^*) + (1 - v)(Q_k - Q^*)/(Q^- - Q^*)$$

Then, based on the concept above, the best situation, when $S^* = 0$ and $S^- = 1$, and the worst situation, when $Q^* = 0$ and $Q^- = 1$, can be rewritten as follow:

$$R_k = vS_k + (1 - v)Q_k$$

The Modified VIKOR method (5)

This research seeks to combine the influential weights of the DANP with the VIKOR method to determine how to minimise the average gap (or regret) and prioritise improvement in the maximum gap overall and in each dimension based on the INRM by the DEMATEL technique. Thus, this study focuses on how to improve and reduce the performance gaps to achieve the aspiration level based on INRM. Please ensure that the intended meaning has been maintained in this edit.



Fuzzy Integral

Hybrid MADM Model
Non-additive / Super-additive
Based concept from Kahneman in 1969S
[Kahneman, 2002 Novel Prize, from experiment]
Kahneman-Tversky (prospect theory)
Von Neumann-Morgeustern (Expected utility model
Fishburn (bilateral independence)
Keeney (Utility independence)
Tzeng (New hybrid MCDM field for Tomorrow)

Fuzzy Integral (1)

- Multiple attribute decision making (MADM) involves
  - Determining the optimal alternative among multiple, conflicting, and interactive criteria (Chen and Hwang, 1992).
- Many methods, which are based on multiple attribute utility theory (MAUT), have been proposed to deal with the MCDM problems
  - E.g. the weighted sum and the weighted product methods

Fuzzy Integral (2)

- The concept of MAUT
  - To aggregate all criteria to a specific uni-dimension (called utility function) to evaluate alternatives.
- Therefore, the main issue of MAUT
  - To find a rational and suitable aggregation operator (fusion operator) which can represent the preferences of the decision-maker.

Fuzzy Integral (3)

- Although many papers have been proposed to discuss the aggregation operator of MAUT (Fishburn, 1970), the main problem of MAUT
  - The assumption of preferential independence (Hillier, 2001; Grabisch, 1995); but in real world, it is a non-additive/super-additive MAUT problem.

[Kahneman, 2002 Novel Proze, from his experiment, he also found "it is a non-additive/super-additive MAUT problem" in 1960S] Von Neumann-Morgeustern

Fuzzy Integral (4)

- Preferential independence can be described as the preference outcome of one criterion over another criterion is not influenced by the remaining criteria.
- However, the criteria are usually interactive in the practical MCDM problems.
- In order to overcome this non-additive problem, the Choquet integral was proposed (Choquet, 1953; Sugeno, 1974).

Fuzzy Integral (5)

- The Choquet integral can represent a certain kind of interaction among criteria using the concept of redundancy and support/synergy.

Fuzzy Integral (6)

- In 1974, Sugeno introduced the concept of fuzzy measure and fuzzy integral
  - Generalizing the usual definition of a measure by
    - Replacing the usual additive property with a weaker requirement
      - I.e. the monotonicity property with respect to set inclusion.

Fuzzy Integral (7)

Definition 3.2.1: Let X be a measurable set that is endowed with pro $\aleph \rightarrow [0,1]$ perties of σ-algebra, where $\aleph$ is all subsets of X. A fuzzy measure g defined on the measurable space $(X, \aleph)$ is a set function g: , which satisfies the following properties: (1) $g(\emptyset) = 0, g(X) = 1$; (2) for all $A, B \in \aleph$, if $A \subseteq B$ then $g(A) \leq g(B)$ (monotonicity).

Fuzzy Integral (8)

As in the above definition, $(X, \aleph, g)$ is said to be a fuzzy measure space. Furthermore, as a consequence of the monotonicity condition, we can obtain: $g(A \cup B) \geq \max\{g(A), g(B)\}$, and $g(A \cap B) \leq \min\{g(A), g(B)\}$.

In the case where $g(A \cup B) = \max\{g(A), g(B)\}$, the set function g is called a possibility measure (Zadeh 1978), and if $g(A \cap B) = \min\{g(A), g(B)\}$, g is called a necessity measure.

Fuzzy Integral (9)

Definition 3.2.2: Let $h = \sum_{i=1}^{n} a_i \cdot 1_{A_i}$ be a simple function, where 1_{A_i} is the characteristic function of the set $A_i \in \aleph, i = 1, \cdots, n$; the sets A_i are pairwise disjoint, and $M(A_i)$ is the measure of A_i . Then the Lebesque integral of h is

$$\int h \cdot dM = \sum_{i=1}^{n} M(A_i) \cdot a_i \ .$$

Fuzzy Integral (10)

Definition 3.3.3 Let $(X, \aleph, g)$ be a fuzzy measure space. The Sugeno integral of a fuzzy measure $g : \aleph \to [0,1]$ with respect to a simple function h is defined by $\int h(x) \circ g(x)$ =

$\bigvee_{i=1}^{n} (h(x_{(i)}) \wedge g(A_{(i)}))$ = $\max_{i} \min \left\{ a_i^{'}, g(A_i^{'}) \right\}$, where

$h(x_{(i)})$ is a linear combination of a characteristic function $1_{A_i^{'}}$ such that $A_1 \subset A_2 \subset \cdots \subset A_n$,and

$A_i^{'} = \{ x \mid h(x) \geq a_i^{'} \}$.

Fuzzy Integral (11)

Definition 3.3.4 Let $(X,\aleph,g)$ be a fuzzy measure space. The Choquet integral of a fuzzy measure $g:\aleph\to[0,1]$ with respect to a simple function h is defined by $\int h(x)\cdot dg \cong \sum_{i=1}^{n}\left[h(x_i)-h(x_{i-1})\right]\cdot g(A_i)$, with the same notions as above, and $h(x_{(0)})=0$.

Fuzzy Integral (12)

Let g be a fuzzy measure which is defined on a power set P(x) and satisfies the definition 3.3.1 as above. The following characteristic is evidently,

$\forall A, B \in P(X), A \cap B = \phi \;\Rightarrow\; g_\lambda(A \cup B) =$

$g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B)$, for $-1 \le \lambda \le \infty$.

Fuzzy Integral (13)

Set $X = \{x_1, x_2, \cdots, x_n\}$, the density of fuzzy measure $g_i = g_\lambda(\{x_i\})$ can be formulated as follows: $g_\lambda(\{x_1, x_2, \cdots, x_n\}) = \sum_{i=1}^{n} g_i + \lambda \sum_{i_1=1}^{n-1} \sum_{i_2=i_1+1}^{n} g_{i_1} \cdot g_{i_2} +$

$\cdots + \lambda^{n-1} \cdot g_1 \cdot g_2 \cdots g_n = \frac{1}{\lambda} \left| \prod_{i=1}^{n} (1 + \lambda \cdot g_i) - 1 \right|$, for

$-1 \leq \lambda \leq \infty$.

Fuzzy Integral (14)

Let h is a measurable set function defined on the fuzzy measurable space $(X, \aleph)$, suppose that $h(x_1) \geq h(x_2) \geq \cdots \geq h(x_n)$, then the fuzzy integral of fuzzy measure $g(\cdot)$ with respect to $h(\cdot)$ can be defined as follows (Ishii & Sugeno, 1985; see Fig. 1).

Fuzzy Integral (15)

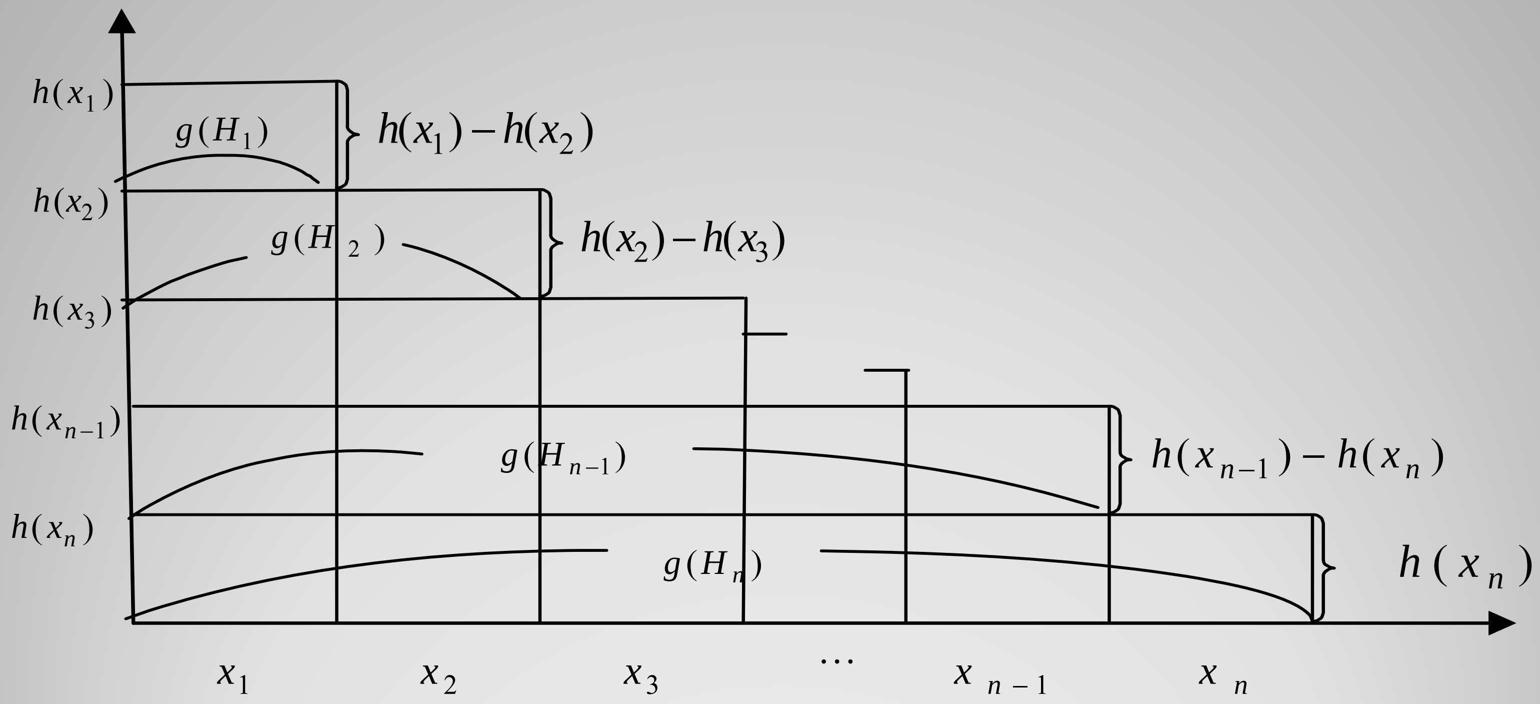


Figure 1 The concept of the Choquet integral

Fuzzy Integral (16)

$\int h \cdot dg = h(x_n) \cdot g(H_n) + [h(x_{n-1}) - h(x_n)] \cdot g(H_{n-1}) + \cdots + [h(x_1) - h(x_2)] \cdot g(H_1) = h(x_n) \cdot [g(H_n) - g(H_{n-1})] + h(x_{n-1}) \cdot [g(H_{n-1}) - g(H_{n-2})] + \cdots + h(x_1) \cdot g(H_1)$, where $H_1 = \{x_1\}$, $H_2 = \{x_1, x_2\}$, $\cdots$, $H_n = \{x_1, x_2, \cdots, x_n\} = X$. In addition, if $\lambda = 0$ and $g_1 = g_2 = \cdots = g_n$ then $h(x_1) \geq h(x_2) \geq \cdots \geq h(x_n)$ is not necessary.

Fuzzy Measure with Variable Additivity Degree (1)

- A fuzzy measure with variable degree of additivity is proposed to overcome the above mentioned problems

Empirical case

Evaluating mobile learning adoption in higher education based on new hybrid MCDM models

In real case
For solving real problems

An empirical case-mobile learning adoption in higher education of Taiwan

- This section presents an empirical case involving Taiwan to emulating mobile learning adoption in higher education based on a new hybrid MCDM model.

Basic concept

- This study investigated the mobile learning adoption of evaluation in higher education. Mobile learning is a new form of learning utilizing the unique of mobile devices. However, students' readiness for mobile learning has yet to fully explore in Taiwan.

Introduction

This study contributes in higher education in three ways.

- First, the adoption of mobile learning is explored from a multi-faceted perspective including: **(1) attitude-related behaviours to mobile learning**, **(2) perceived behavioural control**, **and (3) trust-related behaviours**. This implies that university practitioners should consider these three factors before **employing m-learning**.
- Second, the current study shows the relative importance **of perceived behaviour control** (i.e**., perceptions of internal and external constraints on behaviour)** (Taylor and Todd, 1995) in the decision to adopt mobile learning.
- Lastly, the current findings reveal that **usefulness and ease of use affect students' attitude for adopting mobile learning**. Thus, to facilitate the acceptance of mobile learning, the learning environment should be perceived as useful and easy to use.

Purpose

- The purpose of the present study is to address these issues; we develop a new hybrid MADM model that combines DEMATEL, DANP (DEMATEL-based ANP), and VIKOR.
- The new hybrid method overcome the limitations of existing decision models and can be used to help us analyze the criteria that influence mobile learning issue (relieve and relax some unrealistic assumptions or hypotheses in the real world).
- In particular, we use Taiwan's college students as an example to study the interdependence among the factors that influence the user behavior of mobile learning in the higher education as well as evaluate alternative user behavior processes to achieve the aspired levels of performance from mobile learning.

Framework of dimensions and criteria

Dimensions	Criteria
Attitude-related behaviours D_1	Relative advantage C_1
	Compatibility C_2
	Complexity C_3
Perceived behavioural control D_2	Self-efficacy C_4
	Resource facilitating conditions C_5
	Technology facilitating conditions C_6
Trust-related behaviours D_3	Disposition to trust C_7
	Structural assurance C_8
	Trust belief C_9

Data Collection

- The data was collected from 32 education experts who understand mobile learning trend and usage (in consensus, significant confidence is 96.375%, more than 95%; i.e., gap error =3.265%, smaller less 5%).
- Most of the education experts have teaches more than ten years in higher education.
- Expert perspectives on all criteria within the criteria were collected via personal interviews and a questionnaire.
- Expert elicitation was conducted in Nov., 2012, and it took 60 to70 minutes for each subject to complete a survey.

DEMATEL

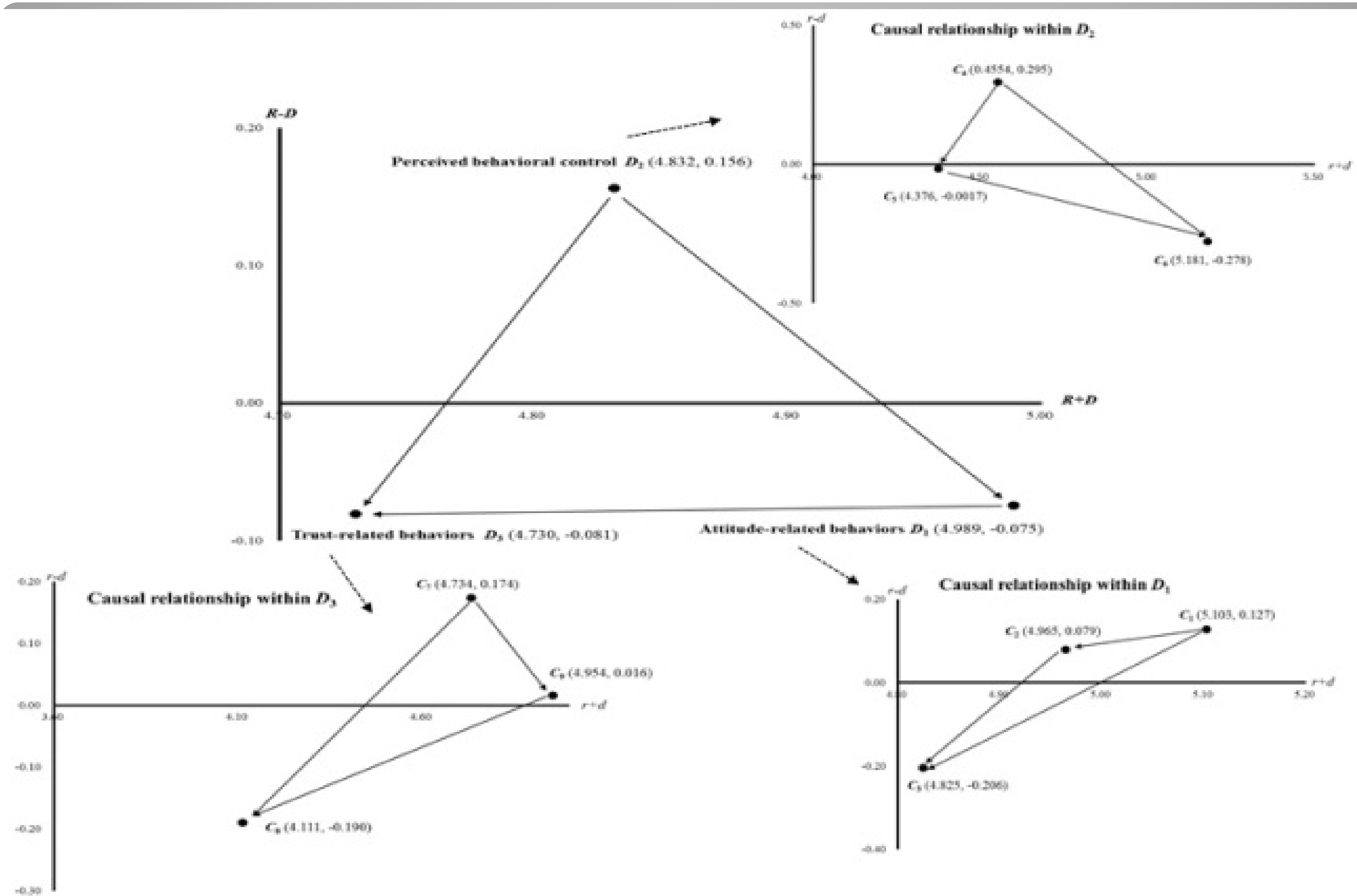
- This study obtained the total influential matrix ***T*** of the dimensions, as shown in Table 1.

D	D_1	D_2	D_3	d_i	s_i	$d_{i+}s_i$	$d_{i-}s_i$
D_1	0.827	0.813	0.817	2.457	2.532	**4.989**	**-0.075**
D_2	0.888	0.784	0.822	2.494	2.338	**4.832**	**0.156**
D_3	**0.817**	**0.741**	**0.767**	**2.325**	**2.406**	**4.730**	**-0.081**

DEMATEL

- This study obtained the total influential matrix ***T*** of the criteria, as shown below.

Dimensions/ Criteria	r_i	d_i	$r_i + d_i$	$r_i - d_i$	Degree of importance (Global weights)	Ranking
Attitude-related behaviors (D_1)					**0.348**	**1**
Relative advantage (C_1)	2.522	2.443	4.965	0.079	0.115	5
Compatibility (C_2)	2.615	2.488	5.103	0.127	0.118	3
Complexity (C_3)	2.310	2.515	4.825	-0.206	0.116	4
Perceived behavioral control (D_2)					**0.322**	**3**
Self-efficacy (C_4)	2.425	2.129	4.554	0.295	0.097	9
Resource facilitating conditions (C_5)	2.179	2.196	4.376	-0.017	0.100	8
Technology facilitating conditions (C_6)	2.451	2.729	5.181	-0.278	0.125	1
Trust-related behaviors (D_3)					**0.331**	**2**
Disposition to trust (C_7)	2.454	2.280	4.734	0.174	0.109	6
Structural assurance (C_8)	1.961	2.150	4.111	-0.190	0.102	7
Trust belief (C_9)	2.485	2.469	4.954	0.016	0.119	2


Causal relationship within D2
C4 (0.4554, 0.295)
C5 (4.376, -0.0017)
C6 (5.181, -0.278)
Perceived behavioral control D2 (4.832, 0.156)
R-D
R+D
Trust-related behaviors D3 (4.730, -0.081)
Attitude-related behaviors D1 (4.989, -0.075)
Causal relationship within D3
C7 (4.734, 0.174)
C8 (4.954, 0.016)
C9 (4.111, -0.190)
Causal relationship within D1
C1 (5.103, 0.127)
C2 (4.965, 0.079)
C3 (4.825, -0.206)

The gap evaluation of mobile learning by DANP & VIKOR

D/C	Local Weight	Global weight (DANP)	Mobile learning gap (r_{kj})
D_1	**0.348**		**0.197**
C_1	0.329	0.115	0.113
C_2	0.339	0.118	0.213
C_3	0.332	0.116	0.266
D_2	**0.322**		**0.296**
C_4	0.300	0.097	0.228
C_5	0.310	0.100	0.366
C_6	0.389	0.125	0.294
D_3	**0.331**		**0.295**
C_7	0.331	0.109	0.266
C_8	0.310	0.102	0.338
C_9	0.359	0.119	0.284
	Total gaps		**0.261**

Sequence of improvement priority for mobile learning user behaviour

Formula	Sequence of improvement priority
F1:Influential network of dimensions	$(D_2),(D_1),(D_3)$ (D_1): $(C_1),(C_2),(C_3)$ (D_2): $(C_4),(C_5),(C_6)$
F2:Influential network of criteria within individual dimensions	$(D_3),(D_2),(D_1)$
F3:Sequence of dimension to rise to aspired/desired level (by gap value, from high to low)	(D_1): $(C_3),(C_2),(C_1)$ (D_2): $(C_5),(C_6),(C_4)$ (D_3): $(C_7),(C_9),(C_8)$
F1:Influential network of dimensions	$(D_2),(D_1),(D_3)$ (D_1): $(C_1),(C_2),(C_3)$ (D_2): $(C_4),(C_5),(C_6)$

Conclusions

- Mobile learning service has an important role in the training of higher education. Its decisions are complicated by the fact that various criteria are uncertainty and may vary across the different product categories and use situations.
- Based on the export and literature review, we developed the three dimensions and 9 criteria that align with the mobile learning service of environment.
- The main reason is among the numerous approaches that are available for conflict management, hybrid MCDM is one of the most prevalent. VIKOR is a method within MCDM; it is based on an aggregating function representing closeness to the ideal (aspiration level), which can be viewed as a derivative of compromise programming for avoiding "choose the best among inferior alternatives (i.e., pick the best apple among a barrel of rotten apples)".

Empirical case

A New Hybrid MADM Model for Problems-Improvement

In real case
For solving real problems

An empirical case-TDC of Taiwan

- This section presents an empirical case involving Taiwan to explore strategies for improving tourism destination competiveness (TDC) based on a new hybrid MCDM model.

Background
(why this topic is the most significant issues?)

- Tourism industry should be considered as a key contributor to Taiwan's overall economic growth.
- World Economic Forum (2009) presented the world Travel & Tourism Competitiveness Index, on which Taiwan ranked 9th in the Asia Pacific and 43th in the world.
- However, few studies have focused on exploring strategies for improving TDC in Taiwan.

Research Purposes

- Exploring strategies for improving tourism destination competitiveness (TDC) in Taiwan based on a new hybrid MCDM model.

Data collection

- A list of dimensions/criteria that can enhance TDC was gathered based on a tourism competitiveness report from World Economic Forum in 2009.
  - **Regulatory framework(D_1)**
    - policy rules and regulations(C_1), environmental sustainability(C_2), safety and security(C_3), health and hygiene(C_4), prioritization of Travel & Tourism(C_5)
  - **Business environment and infrastructure(D_2)**
    - air transport infrastructure(C_6), ground transport infrastructure(C_7), tourism infrastructure(C_8), Information and Communication Technology (ICT) infrastructure(C_9), price competitiveness(C_{10})
  - **Human, cultural, and natural resources(D_3)**
    - human resources(C_{11}), affinity for Travel & Tourism(C_{12}), natural resources(C_{13}), cultural resources(C_{14}).

Data collection

- This study used a four-point scale ranging from 0 (no influence) to 4 (very high influence) to identify the criteria and their influence on one another.
  - **The experts had backgrounds in travel and tourism fields (national and private universities in Taiwan).**
  - **Fifteen experts-the consensus rates of the dimensions and criteria were 96.89% and 96.71% (both exceeding 96% in confidence).**
- This study gathered secondary data on competitiveness score of dimensions and criteria from the tourism competitiveness report published in 2009.

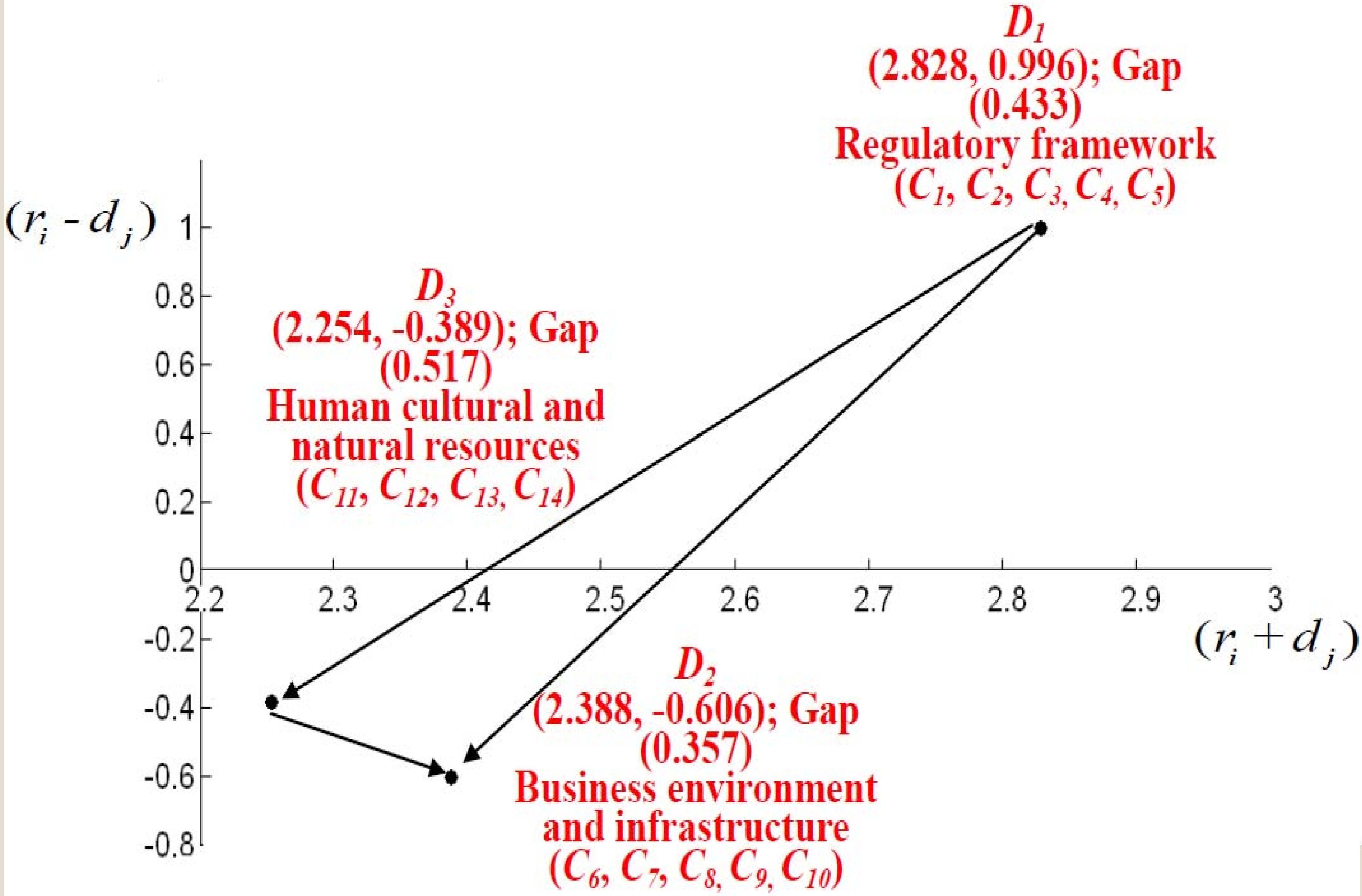
DEMATEL

- This study obtained the total influential matrix T of the dimensions, as shown in Table 1.

Table 1. Total influential matrix of T and the sum of the effects on the dimensions

Dimensions		D_1	D_2	D_3	r_i	d_i	r_i+d_i	r_i-d_i
D_1	Regulatory framework	0.305	0.825	0.782	1.912	0.916	2.828	0.996
D_2	Business environment and infrastructure	0.321	0.237	0.332	0.891	1.497	2.388	-0.606
D_3	Human cultural and natural resources	0.290	0.435	0.208	0.932	1.322	2.254	-0.389

DEMATEL



D_1
(2.828, 0.996); Gap (0.433)
Regulatory framework
(C_1, C_2, C_3, C_4, C_5)
D_3
(2.254, -0.389); Gap (0.517)
Human cultural and natural resources
(C_{11}, C_{12}, C_{13}, C_{14})
D_2
(2.388, -0.606); Gap (0.357)
Business environment and infrastructure
(C_6, C_7, C_8, C_9, C_{10})
$(r_i - d_j)$
$(r_i + d_j)$
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3

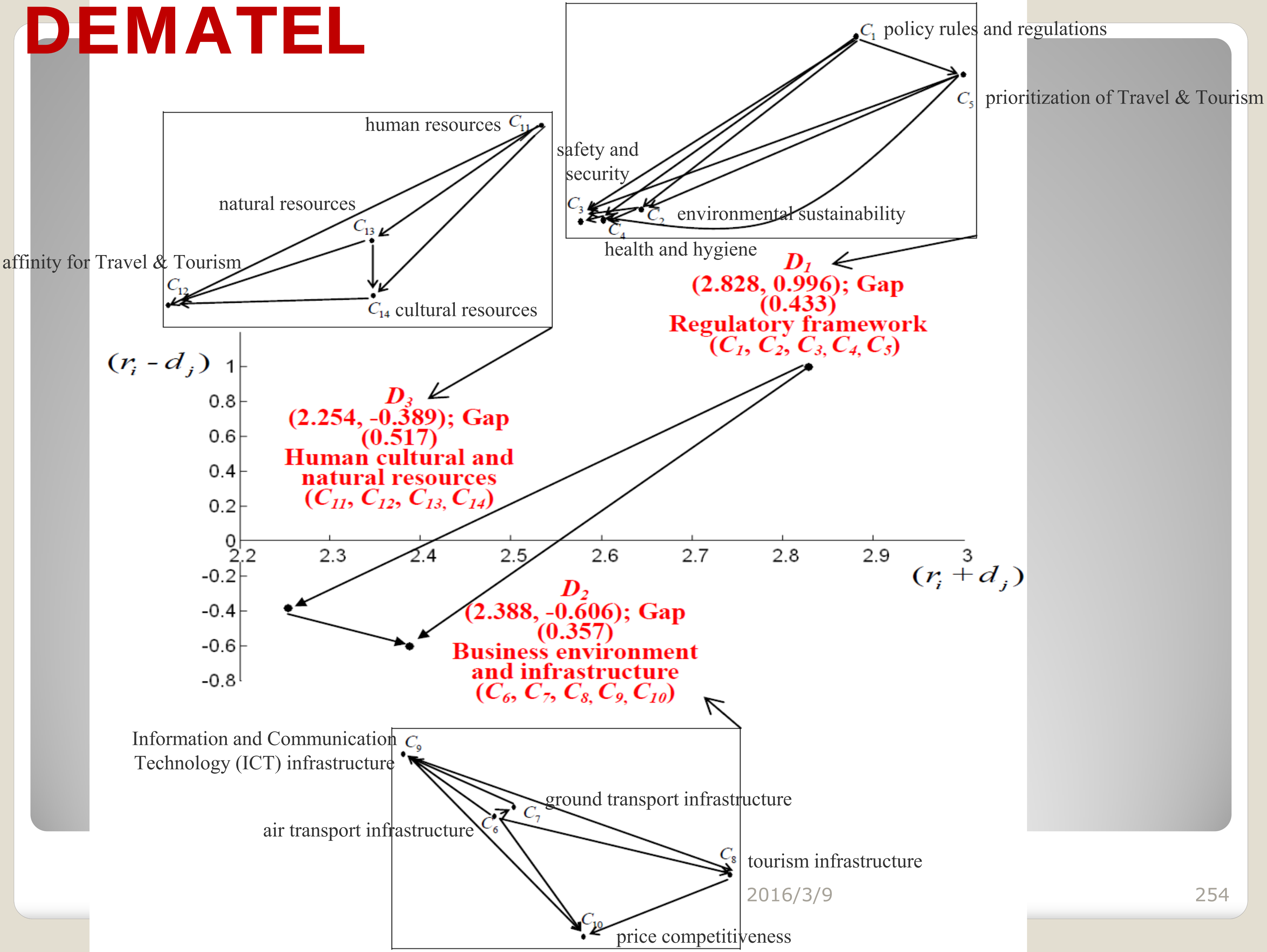
DEMATEL

- This study obtained the total influential matrix *T* of the criteria, as shown in Table 2.

Table 2. The sum of the effects, weights and rankings of each criterion

Criteria	r_i	d_j	$r_i + d_j$	$r_i - d_j$	Degree of importance (Global weight)	Ranking
D_1					0.2866	3
C_1	1.750	0.882	2.633	0.868	0.0544	3
C_2	0.865	0.933	1.798	-0.068	0.0546	2
C_3	0.716	0.846	1.562	-0.131	0.0500	5
C_4	0.764	0.886	1.651	-0.122	0.0537	4
C_5	1.857	1.192	3.048	0.665	0.0739	1
D_2					0.3803	1
C_6	0.726	0.935	1.661	-0.209	0.0744	3
C_7	0.735	0.936	1.670	-0.201	0.0739	4
C_8	0.754	1.020	1.774	-0.266	0.0809	1
C_9	0.734	0.884	1.618	-0.150	0.0717	5
C_{10}	0.690	1.014	1.704	-0.325	0.0794	2
D_3					0.3332	2
C_{11}	1.103	0.778	1.881	0.325	0.0769	4
C_{12}	0.729	0.930	1.659	-0.202	0.0837	3
C_{13}	0.884	0.896	1.780	-0.013	0.0841	2
C_{14}	0.803	0.977	1.781	-0.174	0.0885	1

DEMATEL



C1 policy rules and regulations
C5 prioritization of Travel & Tourism
safety and security
C3
C2 environmental sustainability
C4
health and hygiene
human resources C11
natural resources
C13
affinity for Travel & Tourism
C12
C14 cultural resources
D1
(2.828, 0.996); Gap (0.433)
Regulatory framework
(C1, C2, C3, C4, C5)
D3
(2.254, -0.389); Gap (0.517)
Human cultural and natural resources
(C11, C12, C13, C14)
D2
(2.388, -0.606); Gap (0.357)
Business environment and infrastructure
(C6, C7, C8, C9, C10)
(ri - dj)
(ri + dj)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3
Information and Communication Technology (ICT) infrastructure
C9
ground transport infrastructure
C7
air transport infrastructure
C6
C8
tourism infrastructure
C10
price competitiveness

DANP

- This study builds the assessment model using DEMATEL, which is combined with the DANP (DEMATEL-based ANP) model to obtain the influential weights of each criterion, as shown in Table 2.

Table 2. The sum of the effects, weights and rankings of each criterion

Criteria	r_i	d_j	$r_i + d_j$	$r_i - d_j$	Degree of importance (Global weight)	Ranking
D_1					0.2866	3
C_1	1.750	0.882	2.633	0.868	0.0544	3
C_2	0.865	0.933	1.798	-0.068	0.0546	2
C_3	0.716	0.846	1.562	-0.131	0.0500	5
C_4	0.764	0.886	1.651	-0.122	0.0537	4
C_5	1.857	1.192	3.048	0.665	0.0739	1
D_2					0.3803	1
C_6	0.726	0.935	1.661	-0.209	0.0744	3
C_7	0.735	0.936	1.670	-0.201	0.0739	4
C_8	0.754	1.020	1.774	-0.266	0.0809	1
C_9	0.734	0.884	1.618	-0.150	0.0717	5
C_{10}	0.690	1.014	1.704	-0.325	0.0794	2
D_3					0.3332	2
C_{11}	1.103	0.778	1.881	0.325	0.0769	4
C_{12}	0.729	0.930	1.659	-0.202	0.0837	3
C_{13}	0.884	0.896	1.780	-0.013	0.0841	2
C_{14}	0.803	0.977	1.781	-0.174	0.0885	1

VIKOR

- A real case involving Taiwan is used to assess the total competitiveness using the VIKOR method, as listed in Table 3.
- The scores of each criterion and the total average gap (S_k) of Taiwan are obtained, using the relative influential weights from DANP to multiply the gap(r_{kj}) .

Table 3. The performance evaluation of the case study by VIKOR

Dimensions / Criteria	Local weight	Global weight (by DANP)	Case study of Taiwan	
			Score	Gap (r_{kj})
D_1	**0.2866(3)**		**4.40**	**0.433**
C_1	0.1898	0.0544(3)	4.80	0.367
C_2	0.1905	0.0546(2)	4.20	0.467
C_3	0.1745	0.0500(5)	5.50	0.250
C_4	0.1874	0.0537(4)	3.30	0.617
C_5	0.2579	0.0739(1)	4.20	0.467
D_2	**0.3803(1)**		**4.90**	**0.357**
C_6	0.1956	0.0744(3)	3.80	0.533
C_7	0.1943	0.0739(4)	5.70	0.217
C_8	0.2127	0.0809(1)	4.40	0.433
C_9	0.1885	0.0717(5)	5.30	0.283
C_{10}	0.2088	0.0794(2)	5.10	0.317
D_3	**0.3332(2)**		**3.90**	**0.517**
C_{11}	0.2308	0.0769(4)	5.70	0.217
C_{12}	0.2512	0.0837(3)	4.60	0.400
C_{13}	0.2524	0.0841(2)	2.40	0.767
C_{14}	0.2656	0.0885(1)	2.90	0.683
Total performances			**4.40**	-
Total gap (S_k)			-	**0.437**

Discussions and implications

- Figure 4 shows valuable cues for making correct decisions.
- The influential relation map demonstrate that the degrees of influence among dimensions and criteria.
- This study applies the most important and influential criteria as critical criteria()to improve the maximal gap () of TDC.

DEMATEL

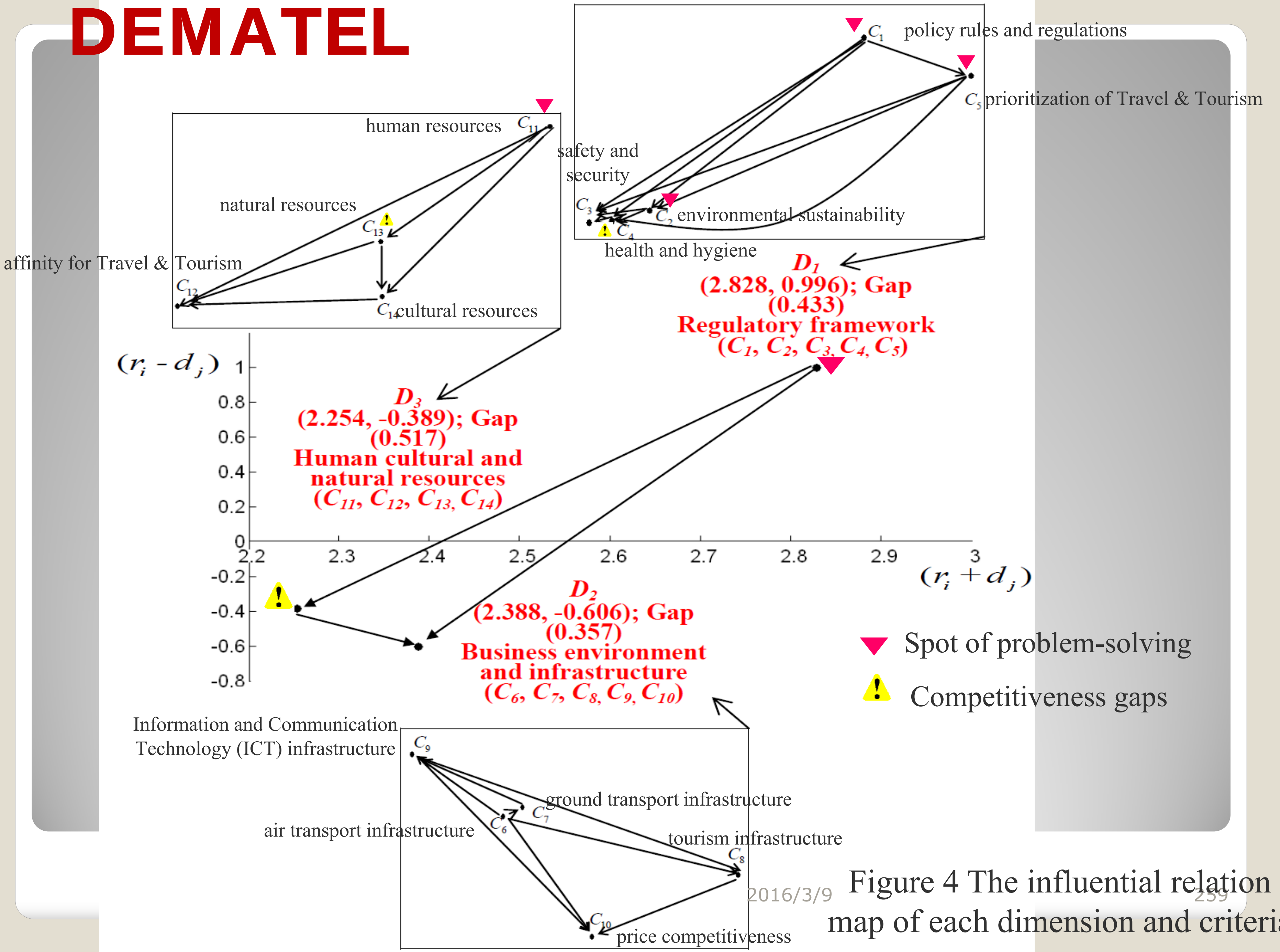


Figure 4 The influential relation map of each dimension and criteria

An empirical case- Conclusions

- This study can obtain valuable cues for making correct decisions to improve TDC.
- This study uses the DEMATEL to develop cause-and-effect influential relationships, calculates the weight using DANP and uses VIKOR method to evaluate competitiveness.
- The decision-maker should improve the cause criteria to successfully improve TDC to achieve the aspiration levels.

An empirical case- Taiwanese company for supplier evaluation and improvement

- This section presents an empirical case involving Taiwanese company for supplier evaluation and improvement based on a novel fuzzy integral-based hybrid MCDM model that addresses the dependence/relationships among the various criteria and the non-additive gap-weighted analysis.

Data collection

- This discussion with the industry helped us to classify the various decision-making criteria into four dimensions (or called perspectives) and 11 criteria.
  - **Compatibility (D_1)**
    - Relationship(C_{11}), Flexibility(C_{12}), Information sharing (C_{13})
  - **Quality (D_2)**
    - Knowledge and skills(C_{21}), Customer satisfaction(C_{22}), On-time rate(C_{23})
  - **Cost (D_3)**
    - Cost saving(C_{31}), Flexibility in billing(C_{32})
  - **Risk (D_4)**
    - Labor union(C_{41}), Loss of management control(C_{42}), Information security(C_{43})

DEMATEL

- Following the DANP procedures, the managers were asked to determine the influence degrees of the relationships among the criteria.
- The sum of the influence given $(r_i - d_j)$ and received $(r_i + d_j)$ for each dimension and criterion (Table 7).

Table 7 Sum of influences given r_i and received d_j on dimensions and criteria

T^D	r_i	d_j	$r_i + d_j$	$r_i - d_j$	T^C	r_i	d_j	$r_i + d_j$	$r_i - d_j$
					C_{11}	3.73	3.61	7.34	0.12
D_1	1.21	1.18	2.39	0.04	C_{12}	3.12	3.02	6.14	0.09
					C_{13}	3.33	3.22	6.55	0.11
					C_{21}	2.43	2.11	4.54	0.33
D_2	0.78	0.89	1.67	-0.11	C_{22}	2.23	2.87	5.10	-0.65
					C_{23}	1.88	2.59	4.48	-0.71
					C_{31}	2.30	2.21	4.51	0.09
D_3	0.76	0.79	1.54	-0.03	C_{32}	1.89	2.17	4.07	-0.28
					C_{41}	3.09	2.76	5.85	0.34
D_4	1.11	1.00	2.12	0.11	C_{42}	3.68	2.96	6.64	0.72
					C_{43}	2.59	2.74	5.33	-0.16

DEMATEL

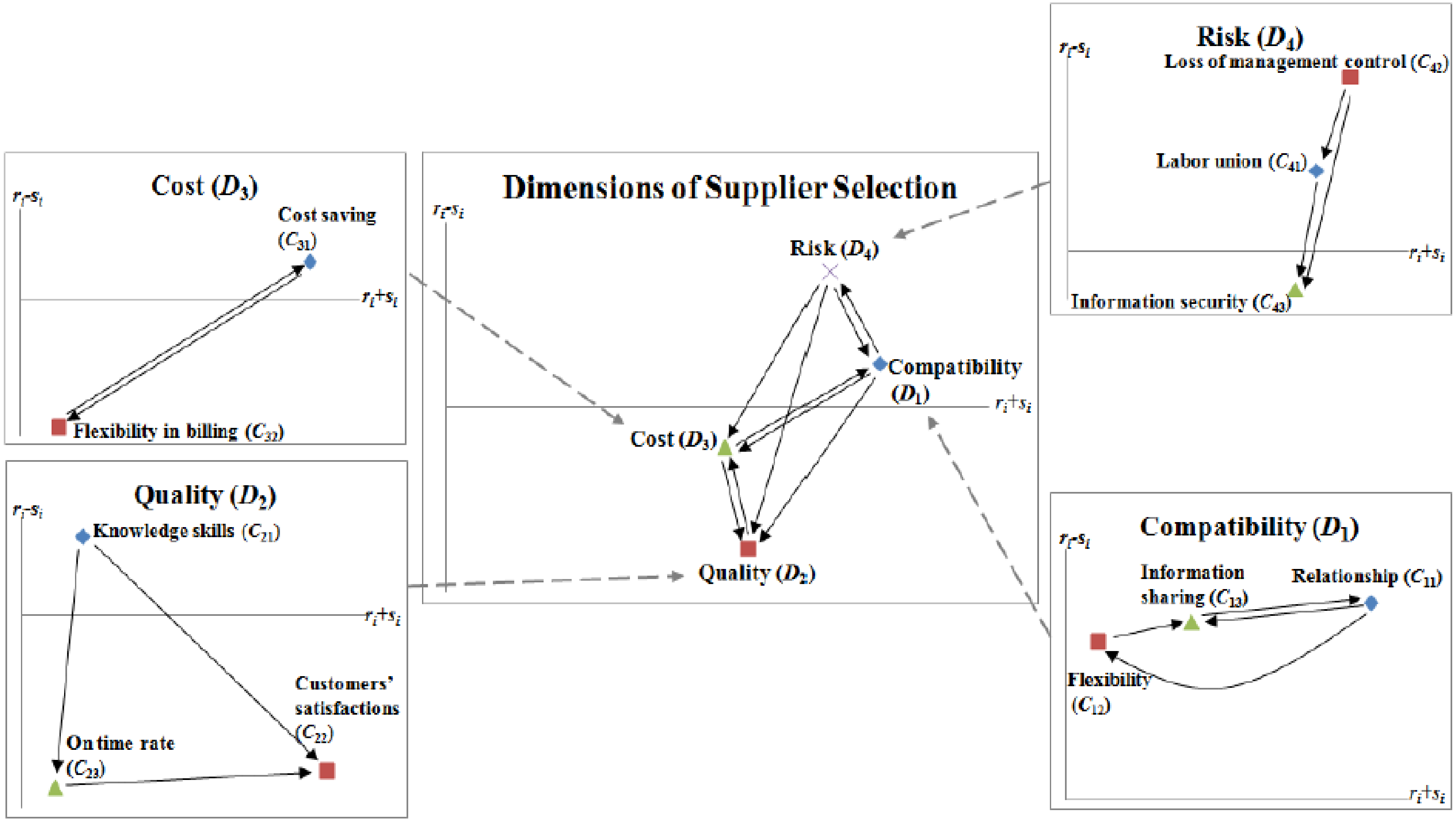


Figure 5 Influential network-relationship map within systems

DANP

- This study builds the assessment model using DEMATEL, which is combined with the DANP (DEMATEL-based ANP) model to obtain the influential weights of each criterion, as shown in Table 8.

Table 8 Influential weights of system factors

Dimensions	Local Weights	Rankings	Criteria	Local Weights	Rankings	Global Weights
			C_{11}	0.367	1	0.112
D_1	0.306	1	C_{12}	0.310	3	0.095
			C_{13}	0.324	2	0.099
			C_{21}	0.281	3	0.065
D_2	0.231	3	C_{22}	0.379	1	0.088
			C_{23}	0.340	2	0.079
D_3	0.204	4	C_{31}	0.506	1	0.103
			C_{32}	0.494	2	0.101
			C_{41}	0.327	2	0.085
D_4	0.259	2	C_{42}	0.351	1	0.091
			C_{43}	0.322	3	0.083

Fuzzy integrals

- This study first transform the performance values into the aspiration level gap values.
- Then, through the obtained global weights and gaps for each criterion and dimension, we synthesize the influential weights and gap values.
- In contrast to previous studies that only apply additive models (i.e., simple additive weight, VIKOR, TOPSIS, grey relation), we utilize fuzzy integrals to aggregate the weighted gaps.

Fuzzy integrals

- Through a questionnaire survey conducted by managers of the case company, the fuzzy integral λ values, which range from -1 to positive infinity , that represent the properties of substitutive or multiplicative between criteria are obtained.
- There are substitutive effects among attributes of risk and there is a multiplicative effect among compatibility, quality, and cost.
- The λ values and the fuzzy measures $g(\cdot)$ are shown in **Table 9**.

Table 9 Fuzzy measure $g(\lambda)$ of each parameter and parameter combination

Fuzzy Measure $g(\cdot)$			
Supplier Selection (evaluating systems) λ = -0.597, q = 1.358			
$g_\lambda(\{D_1\})$ = 0.415	$g_\lambda(\{D_1,D_2\})$ = 0.651	$g_\lambda(\{D_1,D_2,D_3\})$ = 0.821	$g_\lambda(\{D_1,D_2,D_3,D_4\})$ =1
$g_\lambda(\{D_2\})$ = 0.314	$g_\lambda(\{D_1,D_3\})$ = 0.624	$g_\lambda(\{D_1,D_2,D_4\})$ = 0.866	
$g_\lambda(\{D_3\})$ = 0.277	$g_\lambda(\{D_1,D_4\})$ = 0.680	$g_\lambda(\{D_1,D_3,D_4\})$ = 0.844	
$g_\lambda(\{D_4\})$ = 0.352	$g_\lambda(\{D_2,D_3\})$ = 0.539	$g_\lambda(\{D_2,D_3,D_4\})$ = 0.778	
	$g_\lambda(\{D_2,D_4\})$ = 0.600		
	$g_\lambda(\{D_3,D_4\})$ = 0.571		
Compatibility (D_1) λ = 0.358 , q = 0.900			
$g_\lambda(\{C_{11}\})$ = 0.330	$g_\lambda(\{C_{11},C_{12}\})$ = 0.642	$g_\lambda(\{C_{11},C_{12},C_{13}\})$ = 1	
$g_\lambda(\{C_{12}\})$ = 0.279	$g_\lambda(\{C_{11},C_{13}\})$ = 0.656		
Quality (D_2) λ = 3.902, q = 0.539			
$g_\lambda(\{C_{21}\})$ = 0.151	$g_\lambda(\{C_{21},C_{22}\})$ = 0.476	$g_\lambda(\{C_{21},C_{22},C_{23}\})$ = 1	
$g_\lambda(\{C_{22}\})$ = 0.204	$g_\lambda(\{C_{21},C_{23}\})$ = 0.443		
$g_\lambda(\{C_{23}\})$ = 0.183	$g_\lambda(\{C_{22},C_{23}\})$ = 0.533		
Cost (D_3) λ = 1.268, q = 0.798			
$g_\lambda(\{C_{31}\})$ = 0.403	$g_\lambda(\{C_{31},C_{32}\})$ = 1		
$g_\lambda(\{C_{33}\})$ = 0.395			
Risk (D_4) λ = -0.073, q = 1.025			
$g_\lambda(\{C_{41}\})$ = 0.336	$g_\lambda(\{C_{41},C_{42}\})$ = 0.687	$g_\lambda(\{C_{41},C_{42},C_{43}\})$ = 1	
$g_\lambda(\{C_{42}\})$ = 0.360	$g_\lambda(\{C_{41},C_{43}\})$ = 0.657		
$g_\lambda(\{C_{43}\})$ = 0.330	$g_\lambda(\{C_{42},C_{43}\})$ = 0.681		

Fuzzy integrals

- Using the obtained $g(\cdot)$ and the original data (**Appendix**, **Table A**), we can obtain the gap-ratios $r_{kj} = (|f_j^* - f_{kj}|)/(|f_j^* - f_j^-|)$ for alternatives $k = 1,2,\ldots,m$, respective to each criterion (**Table 10)**.

Table 10 Gap ratio values of potential suppliers by SAW

Criteria	Weights (Global)	Weights (Local)	Alternative A_1	A_2	A_3	A_4	A_5
Compatibility (D_1)		**0.306**	**0.241**	**0.198**	**0.197**	**0.183**	**0.264**
Relationship (C_{11})	0.112	0.367	0.264	0.208	0.199	0.198	0.268
Flexibility (C_{12})	0.095	0.310	0.214	0.211	0.198	0.176	0.264
Information sharing (C_{13})	0.099	0.324	0.242	0.175	0.194	0.173	0.258
Quality (D_2)		**0.231**	**0.290**	**0.231**	**0.236**	**0.236**	**0.221**
Knowledge skills (C_{21})	0.065	0.281	0.280	0.221	0.275	0.224	0.214
Customer satisfaction (C_{22})	0.088	0.379	0.286	0.255	0.227	0.265	0.203
On time rate (C_{23})	0.079	0.340	0.302	0.213	0.213	0.214	0.246
Cost (D_3)		**0.204**	**0.243**	**0.306**	**0.330**	**0.343**	**0.268**
Cost saving (C_{31})	0.103	0.506	0.246	0.333	0.313	0.324	0.267
Flexibility in billing (C_{32})	0.101	0.494	0.239	0.278	0.348	0.362	0.269
Risk (D_4)		**0.259**	**0.251**	**0.244**	**0.227**	**0.248**	**0.277**
Labor unions (C_{41})	0.085	0.327	0.257	0.292	0.214	0.219	0.275
Loss of management control (C_{42})	0.091	0.351	0.255	0.208	0.218	0.248	0.288
Information security (C_{43})	0.083	0.322	0.242	0.235	0.249	0.278	0.268
Total Gap (rank)			**0.255** (4)	**0.240** (1)	**0.241** (2)	**0.245** (3)	**0.258** (5)

Note: For example alternative A_1, D_1: $(0.264 \times 0.367) + (0.214 \times 0.310) + (0.242 \times 0.324) = 0.241$, and total gap ratio $= 0.241 \times 0.304 + 0.290 \times 0.231 + 0.243 \times 0.204 + 0.251 \times \ 0.259 = 0.225$ (additive); the original data are shown in the **Appendix,** Table A. The gap ratio is $r_{kj} = (|f_j^* - f_{kj}|)/(|f_j^* - f_j^-|)$ for alternatives $k = 1,2,\ldots,m$ and criteria j=1,2,…n.

Fuzzy integrals

- The integrated weighted gaps of each potential supplier are then calculated as shown in **Table 11**.

Table 11 Gap ratio values of potential suppliers by Fuzzy Integral

Criteria	Weights	Alternative				
	Local	A_1	A_2	A_3	A_4	A_5
Compatibility (D_1)	**0.306**	**0.240**	**0.179**	**0.197**	**0.182**	**0.263**
Relationship (C_{11})	0.367	0.264	0.208	0.199	0.198	0.268
Flexibility (C_{12})	0.310	0.214	0.211	0.198	0.176	0.264
Information sharing (C_{13})	0.324	0.242	0.175	0.194	0.173	0.258
Quality (D_2)	**0.231**	**0.286**	**0.224**	**0.227**	**0.227**	**0.214**
Knowledge skills (C_{21})	0.281	0.280	0.221	0.275	0.224	0.214
Customer satisfaction (C_{22})	0.379	0.286	0.255	0.227	0.265	0.203
On time rate (C_{23})	0.340	0.302	0.213	0.213	0.214	0.246
Cost (D_3)	**0.204**	**0.242**	**0.300**	**0.327**	**0.339**	**0.268**
Cost saving (C_{31})	0.506	0.246	0.333	0.313	0.324	0.267
Flexibility in billing (C_{32})	0.494	0.239	0.278	0.348	0.362	0.269
Risk (D_4)	**0.259**	**0.252**	**0.245**	**0.227**	**0.249**	**0.277**
Labor unions (C_{41})	0.327	0.257	0.292	0.214	0.219	0.275
Loss of management control (C_{42})	0.351	0.255	0.208	0.218	0.248	0.288
Information security (C_{43})	0.322	0.242	0.235	0.249	0.278	0.268
Total gap (rank)	-	**0.359** (3)	**0.350** (2)	**0.345** (1)	**0.361** (4)	**0.376** (5)

Note: For example Alternative A_1, D_1: (0.264-0.242) × 0.330)+(0.242-0.214) × 0.656)+(0.214 × 1)=0.240,
total ratio gap: (0.286-0.252) × 0.314)+(0.252-0.242) × 0.600)+(0.242-0.240) × 0.778)+(0.240 × 1) = 0.359 (non-additive)

Fuzzy integrals

- The results of comparison between non-additive and additive methods are illustrated in Table 12.

Table 12 Results comparison between non-additive and additive methods

	Dimension (Additive / Non-Additive)				
	A_1	A_2	A_3	A_4	A_5
D_1 Compatibility $\lambda = 0.358$	0.241 / 0.240 (-1%)	0.198 / 0.179 (-10%)	0.197 / 0.197 (0%)	0.183 / 0.182 (0%)	0.264 / 0.263 (0%)
D_2 Quality $\lambda = 3.902$	0.290 / 0.286 (-1%)	0.237 / 0.231 (-3%)	0.236 / 0.227 (-4%)	0.236 / 0.227 (-4%)	0.221 / 0.214 (-3%)
D_3 Cost $\lambda = 1.268$	0.243 / 0.242 (0%)	0.306 / 0.300 (-2%)	0.330 / 0.327 (-1%)	0.343 / 0.339 (-1%)	0.268 / 0.268 (0%)
D_4 Risk $\lambda = -0.073$	0.251 / 0.252 (1%)	0.244 / 0.245 (1%)	0.227 / 0.227 (0%)	0.248 / 0.249 (1%)	0.277 / 0.277 (0%)
Total gaps $\lambda = -0.597$	0.255 / 0.359 (40%)	0.243 / 0.350 (44%)	0.241 / 0.345 (42%)	0.245 / 0.361 (48%)	0.258/ 0.376 (46%)

Note. Parenthesis represents the increased gap ratio %

New concepts and trends of hybrid MCDM model for Tomorrow: Some examples for the real cases

- Dominance-based rough set approach (DRSA) MCDM
- MADM: DEMATEL, DANP (DEMATEL-based ANP), Integration (Additive: SAW, VIKOR, Grey Relation Analysis, PROMETHEE, ELECTRE; Non-additive: Fuzzy Integral)
- MODM: Changeable Spaces Programming

MODM (Multiple Objective Decision Making

- Tzeng classify **MCDM problems** into three main categories: **multiple rule-based decision making (MRDM), multiple attribute decision making (MADM**), and **multiple objective decision making (MODM))** based on the different purposes and the different data types in interrelationship.
- **MODM** is especially **suitable for the design/planning**, which is to achieve the **best or called aspired goals** (**aspiration level**) by considering the various interactions within the given constrains, **how relax or relieve the given constrains through innovation and creativity** so **that both decision and objective spaces are changeable in our new concepts of.**

MODM (Multiple Objective Decision Making

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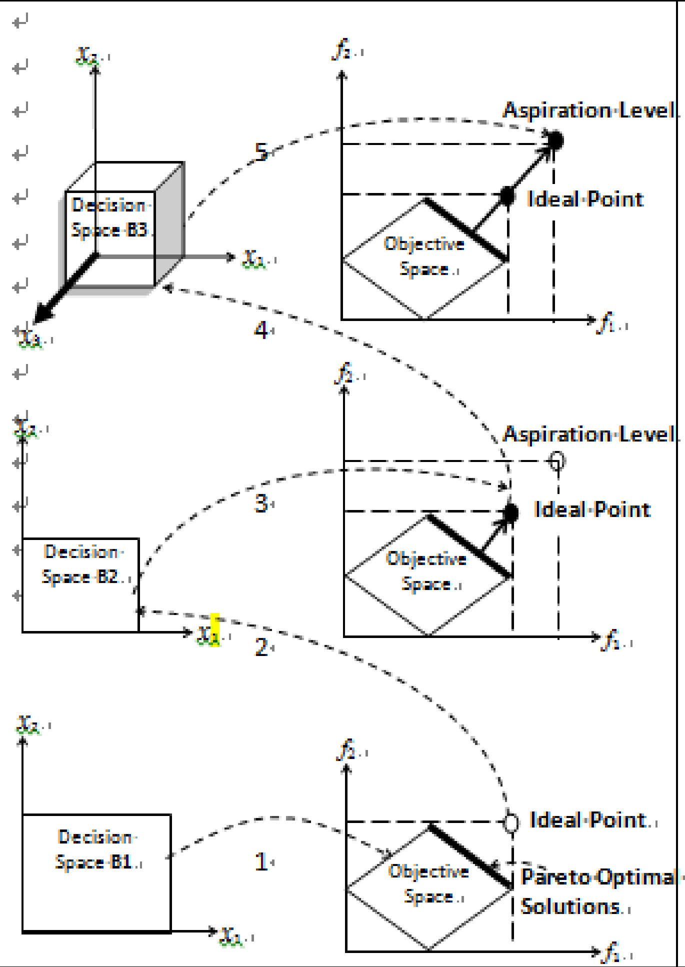
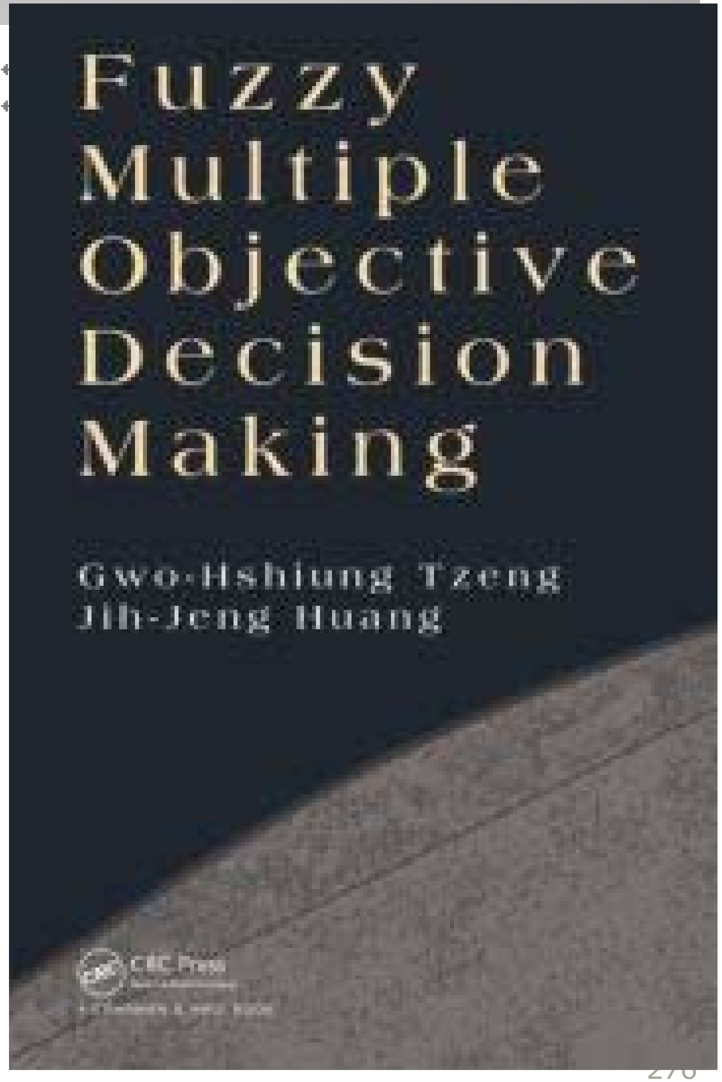
Changeable Spaces Programming

Concept	Graphical Representation	Approach
Value (Win-Win)		making aspired decisions by expanding competence sets through innovation.
		Making Ideal decisions through re-allocating limited resources.
Price (Win-Lose)		Making Pareto optimal decisions through traditional MOP methods



Jih-Jeng Huang, Gwo-Hshiung Tzeng (Corresponding author) (2013), **New thinking of multi-objective programming with changeable space - In search of excellence**, ***Technological and Economic Development of Economy***, **Accepted** (forcoming, **SSCI, IF: 5.605, 2011; IF: 3.235, 2012).**

Gwo-HshiungTzeng, Kuan-Wei Huang, Ching-Wei Lin, and Benjamin J. C. Yuan (2014), **New idea of multi-objective programming with changeable spaces for improving the unmanned factory planning**, PICMET 2014.

Resources reallocation problem

- Pareto Optimal Solutions

$$\text{Max } \{ z_k = \boldsymbol{c}_k \boldsymbol{x} \mid k = 1, \ldots, q \}$$

$$s.t. \ \ \boldsymbol{Ax} \le \boldsymbol{b} \ \rightarrow \ \boldsymbol{pAx} \le \boldsymbol{pb} \ \rightarrow \boldsymbol{vx} \le B \ \ (B \text{ is total budget})$$

$$\boldsymbol{x} \ge 0,$$

- Ideal Point solution (De Novo Programming)

$$\text{Min } \ B = \boldsymbol{vx}$$

$$s.t. \ \ \boldsymbol{c}_k \boldsymbol{x} \ge z_k^* \ (\text{Ideal point}), \ k = 1, \ldots, q$$

$$\boldsymbol{x} \ge 0$$

Resources reallocation problem

- Aspiration level (Changeable spaces programming)

$$\text{Min } \boldsymbol{v}'\boldsymbol{x}$$

$$s.t. \quad \boldsymbol{c}'_{k'}\boldsymbol{x} \geq z^{**}_{k'} \text{ (Aspiration level)}, \; k' = 1, \ldots, q'; \; q' \geq q$$

$$\boldsymbol{x} \geq \boldsymbol{0}$$

where $\boldsymbol{pA}^{*}\boldsymbol{x} \leq \boldsymbol{pb} \rightarrow \boldsymbol{v}'\boldsymbol{x} \leq B \; \left(B \text{ is total budget}\right)$

change technological coefficients in efficiency (Resource Requirement)

Resource Allocation of Zeleny's Example

Table 1 Resource allocation of Zeleny's example.

Unit price	Resource	Technological coefficients $x_1 = 1$	$x_2 = 1$	No. of units
30	Nylon	4	0	20
40	Velvet	2	6	24
9.5	Silver thread	12	4	60
20	Silk	0	3	10.5
10	Golden thread	4	4	26

De Novo Programming Method

- The costs of the given resources portfolio:

$$(30\times 20)+(40\times 24)+(9.5\times 60)+(20\times 10.5)$$
$$+(10\times 26)=\$2600$$

- Unit costs of producing one unit of each of the two products:

$$x_1 \Rightarrow (30\times 4)+(40\times 2)+(9.5\times 12)+(20\times 0)+(10\times 4)=\$354$$
$$x_2 \Rightarrow (30\times 0)+(40\times 6)+(9.5\times 4)+(20\times 3)+(10\times 4)=\$378$$

- Expected profit margins (price-cost) are:

$$x_1 = 754 - 354 = \$400 \,/\, \text{unit}$$
$$x_2 = 678 - 378 = \$3\,00 \,/\, \text{unit}$$

Decision Space and Objective Space

$$\max \quad f_1 = 400x_1 + 300x_2$$
$$\max \quad f_2 = 6x_1 + 8x_2$$
$$s.t. \quad 4x_1 \le 20,$$
$$2x_1 + 6x_2 \le 24,$$
$$12x_1 + 4x_2 \le 60,$$
$$3x_2 \le 10.5,$$
$$4x_1 + 4x_2 \le 26,$$
$$x_1, x_2 \ge 0$$

Objective Space

Decision Space

De Novo Programming Method

- Maximizing total value of function f_1:

$$f_1 = 400x_1 + 300x_2$$

- Maximizing total quality index f_2:

$$f_2 = 6x_1 + 8x_2$$

De Novo Programming Method

- Maximizing levels of two products can be calculated by mathematical programming:

$$
\begin{aligned}
\max \quad & f_1 = 400x_1 + 300x_2 \\
\max \quad & f_2 = 6x_1 + 8x_2 \\
s.t. \quad & 4x_1 \le 20 \\
& 2x_1 + 6x_2 \le 24 \\
& 12x_1 + 4x_2 \le 60 \\
& 3x_2 \le 10.5 \\
& 4x_1 + 4x_2 \le 26 \\
& x_1, x_2 \ge 0
\end{aligned}
$$

- Maximum f_1 in profit:

$$\max \quad f_1 \rightarrow x_1 = 4.25, x_2 = 2.25; \quad f_1^* = 400 \times 4.25 + 300 \times 2.25 = \$2375$$

- Maximum f_2 in total quality index

$$max \quad f_2 \rightarrow x_1 = 3.75, x_2 = 2.75; \quad f_2^* = 6 \times 3.75 + 8 \times 2.75 = \$44.5$$

De Novo Programming Method

- Minimizing the total cost by considering the following constraints:

$$\min \quad 354x_1 + 378x_2$$
$$s.t. \quad f_1 = 400x_1 + 300x_2 \geq 2375$$
$$f_2 = 354x_1 + 378x_2 \geq 44.5$$

- Maximum f_1 in profit:

$$\max \ f_1 \rightarrow x_1 = 4.03, x_2 = 2.54; \quad f_1^* = 400 \times 4.03 + 300 \times 2.54 = \$2375$$

- Maximum f_2 in total quality index:

$$\max \quad f_2 \rightarrow x_1 = 4.03, x_2 = 2.54; \quad f_2^* = 6 \times 4.03 + 8 \times 2.54 = \$44.5$$

- Cost of the newly designed system:

$$(30 \times 16.12) + (40 \times 23.3) + (9.5 \times 58.52) + (20 \times 7.62)$$
$$+ (10 \times 26.28) = \$2386.74$$

De Novo Programming Method

- The new portfolio of resources proposed by the consultant is as following:

Unit price $	Resources (Raw material)	Technological coefficients (Resource Requirement) x_1	x_2	No. of units (Resource portfolio)
30	Nylon	4	0	16.12
40	Velvet	2	6	23.3
9.5	Silver thread	12	4	58.52
20	Silk	0	3	7.62
10	Golden thread	4	4	26.28

- Pareto optimal solutions B=$2600
- De Novo programming, ideal point solution B^*=$2386.74, $B^* < B$.

Modified Example to Demonstrate Yu's Model

Table 2 Modified example to demonstrate Yu's model.

Resource	Technological coefficients $x_1 = 1$	Technological coefficients $x_2 = 1$	No. of units	Unit Purchase Benefit
Nylon	4	0	20	0.3
Velvet	2	6	24	0.3
Silver thread	12	4	60	0.3
Silk	0	3	10.5	0.3
Golden thread	4	4	26	0.3

New Decision Space and Objective Space

$$\max \quad f_1 = 400x_1 + 300x_2 + y(3x_1 + 4x_2)$$

$$\max \quad f_2 = 6x_1 + 8x_2 + y(0.3x_1 + 0.2x_2)$$

$$s.t. \quad 4x_1 \le 20 + 0.3z,$$

$$2x_1 + 6x_2 \le 24 + 0.3z,$$

$$12x_1 + 4x_2 \le 60 + 0.3z,$$

$$3x_2 \le 10.5 + 0.3z,$$

$$4x_1 + 4x_2 \le 26 + 0.3z,$$

$$0 \le y, z \le 7,$$

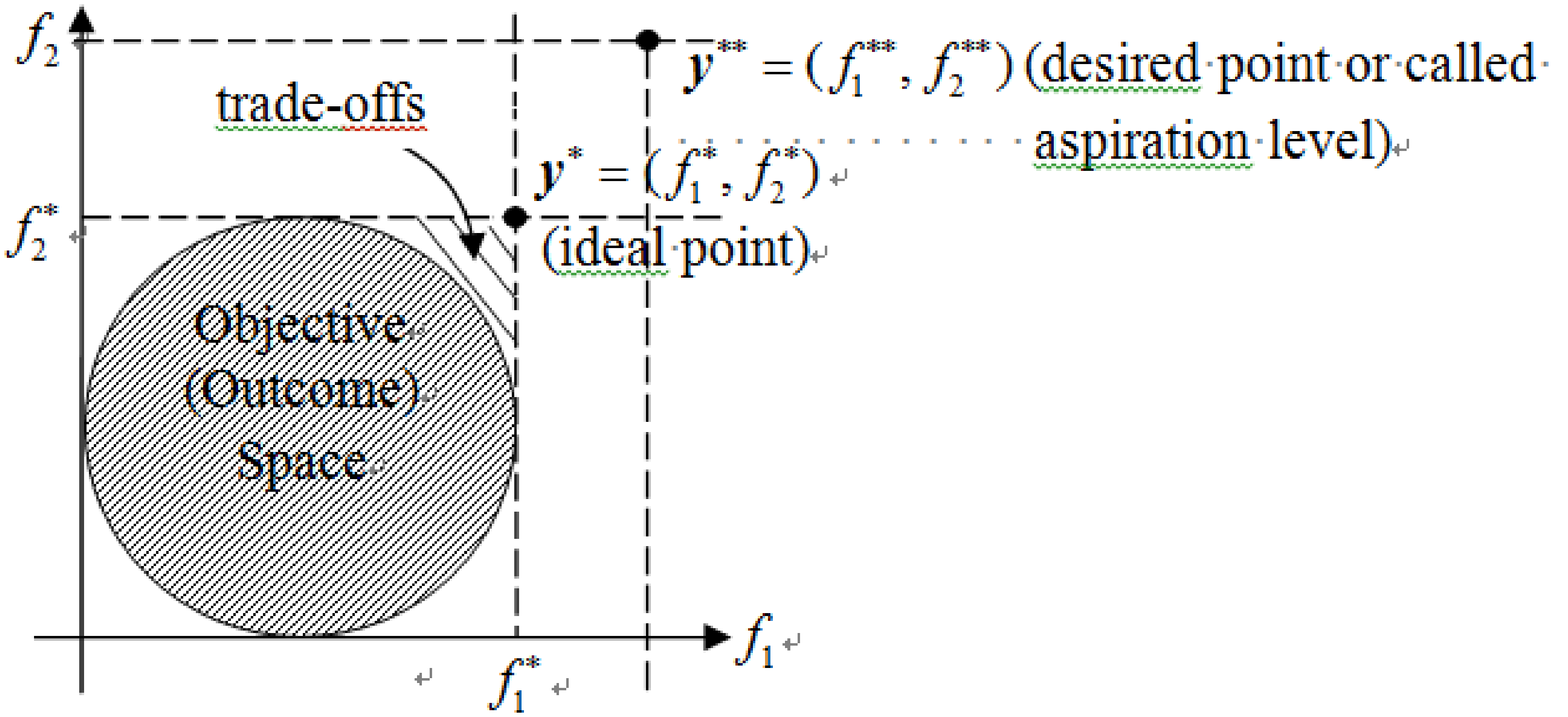
$$y + z \le 10,$$

$$x_1, x_2, y, z \ge 0.$$

Objective Space

Decision Space

Basic concept of the desired point or called aspiration level



MOP with changeable parameters

<Model 1: MOP with changeable budgets>

$$\min \quad \widehat{B} \tag{10}$$

$$s.t. \quad \sum_{j=1}^{m} c_{ij} x_{ij} = f_i^{**}(\boldsymbol{x}), \quad i = 1, \ldots, n,$$

$$\boldsymbol{p}' A \boldsymbol{x} \leq B + \widehat{B},$$

<extra conditions for $\widehat{B}$>

$$\boldsymbol{x} \geq \boldsymbol{0},$$

Information Table for Example 1

Table 3 Information Table for Example 1.

Unit price	Resource	Technological coefficients $x_1 = 1$	$x_2 = 1$	No. of units
30	Nylon	4	0	b_1
40	Velvet	2	6	b_2
9.5	Silver thread	12	4	b_3
20	Silk	0	3	b_4
10	Golden thread	4	4	b_5

Solving the problem of Example 1

$$
\begin{aligned}
\min \quad & \hat{B} \\
s.t. \quad & 400x_1 + 300x_2 = 2600, \\
& 6x_1 + 8x_2 = 60, \\
& 30\times 4x_1 + 40\times(2x_1 + 6x_2) + 9.5\times(12x_1 + 4x_2) + 20\times 3x_2 \\
& +10\times(4x_1 + 4x_2) \le 2600 + \hat{B}, \\
& x_1, x_2 \ge 0.
\end{aligned}
$$

MOP with changeable objective coefficient

<Model 2: MOP with changeable objective coefficients>

$$\min \quad \widehat{B} \qquad (11)$$

$$s.t. \quad \sum_{j=1}^{m} (c_{ij} + \widehat{c}_{ij}) x_{ij} = f_i^{**}(\boldsymbol{x}), \quad i = 1, \ldots, n,$$

$$\boldsymbol{p}'A\boldsymbol{x} + \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij}^{c} \widehat{c}_{ij} \leq B + \widehat{B},$$

<extra conditions for p_{ij}^{c} and $\widehat{c}_{ij}$>

$$\boldsymbol{x} \geq \boldsymbol{0},$$

Information Table for Example

Table 4. Information Table for Example 2.

Objective coefficients $x=1$	Objective coefficients $y=1$	Unit price	Resource	Technological coefficients $x_1=1$	Technological coefficients $x_2=1$	No. of units
400 ($0.200)	300 ($0.289)	30	Nylon	4	0	b_1
6 ($2.225)	8 ($2.487)	40	Velvet	2	6	b_2
		9.5	Silver thread	12	4	b_3
		20	Silk	0	3	b_4
		10	Golden thread	4	4	b_5

Mathematical programming to consider achieving the desired points via improving the objective coefficients

$$
\begin{aligned}
\min \quad & \hat{B} \\
s.t. \quad & (400+\hat{c}_{11})x_1 + (300+\hat{c}_{12})x_2 = 2600, \\
& (6+\hat{c}_{21})x_1 + (8+\hat{c}_{22})x_2 = 60, \\
& 30\times 4x_1 + 40\times(2x_1+6x_2) + 9.5\times(12x_1+4x_2) + 20\times 3x_2 \\
& +10\times(4x_1+4x_2) + (0.200\hat{c}_{11} + 0.289\hat{c}_{12} + 2.225\hat{c}_{21} + 2.487\hat{c}_{22}) \le 2600 + \hat{B}, \\
& x_1, x_2 \ge 0.
\end{aligned}
$$

MOP with Changeable Technological Coefficients

<Model 3: MOP with Changeable technological coefficients >

$$\min \quad \widehat{B} \qquad (12)$$

$$s.t. \quad \sum_{j=1}^{m} c_{ij} x_{ij} = f_i^{**}(\boldsymbol{x}), \quad i = 1, \ldots, n,$$

$$\boldsymbol{p}'(A - \widehat{A})\boldsymbol{x} + \sum_{k=1}^{r} \sum_{j=1}^{m} p_{kj}^{a} \widehat{a}_{kj} \leq B + \widehat{B},$$

<extra conditions for p_{ij}^{a} and $\widehat{a}_{ij}$ >

$$\boldsymbol{x} \geq \boldsymbol{0},$$

Information Table for Example

Table 5. Information Table for Example 3.

Objective coefficient $x=1$	Objective coefficient $y=1$	Unit price	Resource	Technological coefficients $x_1=1$	Technological coefficients $x_2=1$	No. of units
400	300	30	Nylon	4 (\$0.5)	0	b_1
6	8	40	Velvet	2 (\$0.5)	6 (\$0.27)	b_2
		9.5	Silver thread	12 (\$0.27)	4 (\$0.26)	b_3
		20	Silk	0	3 (\$0.25)	b_4
		10	Golden thread	4 (\$0.25)	4 (\$0.25)	b_5

Incorporating the information of the unit updating cost of the technological coefficients

$$\min \quad \hat{B}$$

$$
\begin{aligned}
s.t. \quad & 400x_1 + 300x_2 = 2600, \\
& 6x_1 + 8x_2 = 60, \\
& 30\times(4-\hat{a}_{11})x_1 + 40\times((2-\hat{a}_{21})x_1 + (6-\hat{a}_{22})x_2) + 9.5\times((12-\hat{a}_{31})x_1 \\
& +(4-\hat{a}_{32})x_2) + 20\times(3-\hat{a}_{42})x_2 + 10\times((4-\hat{a}_{51})x_1 + (4-\hat{a}_{52})x_2) \\
& +0.5\hat{a}_{11} + 0.5\hat{a}_{21} + 0.27\hat{a}_{22} + 12\hat{a}_{31} + 4\hat{a}_{32} + 3\hat{a}_{42} + 4\hat{a}_{51} + 4\hat{a}_{52} \le 2600 + \hat{B}, \\
& x_1, x_2 \ge 0.
\end{aligned}
$$

A more general model of changeable parameters

$$\min \quad \hat{B} \qquad (13)$$

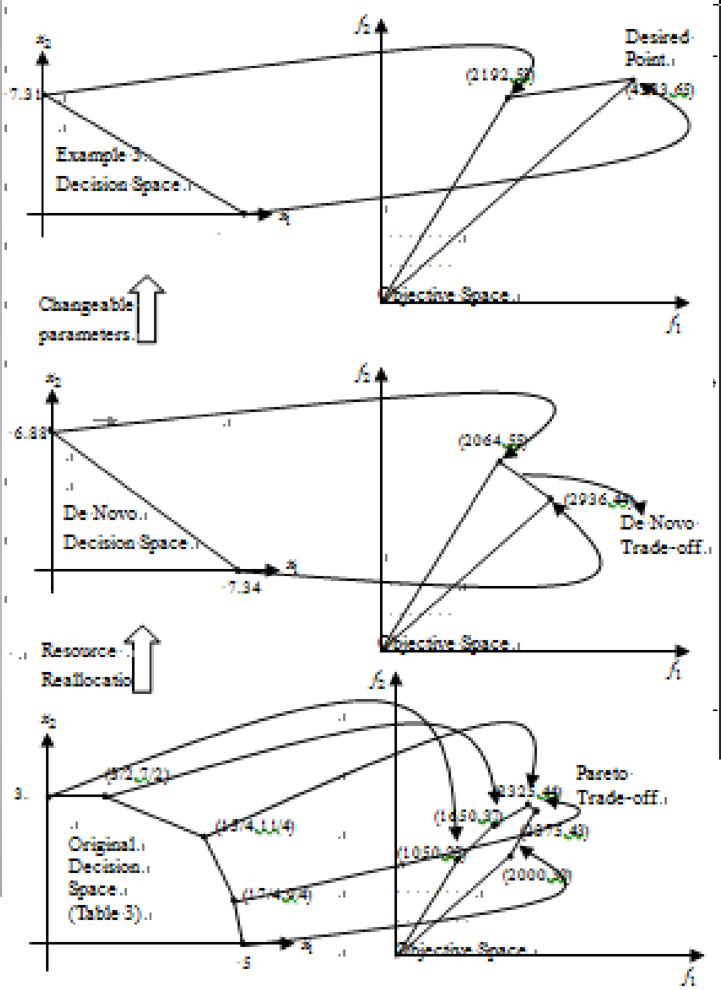
$$s.t. \quad \sum_{j=1}^{m}(c_{ij}+\hat{c}_{ij})x_j \geq f_i^{**}(\boldsymbol{x}), \quad i=1,\ldots,n,$$

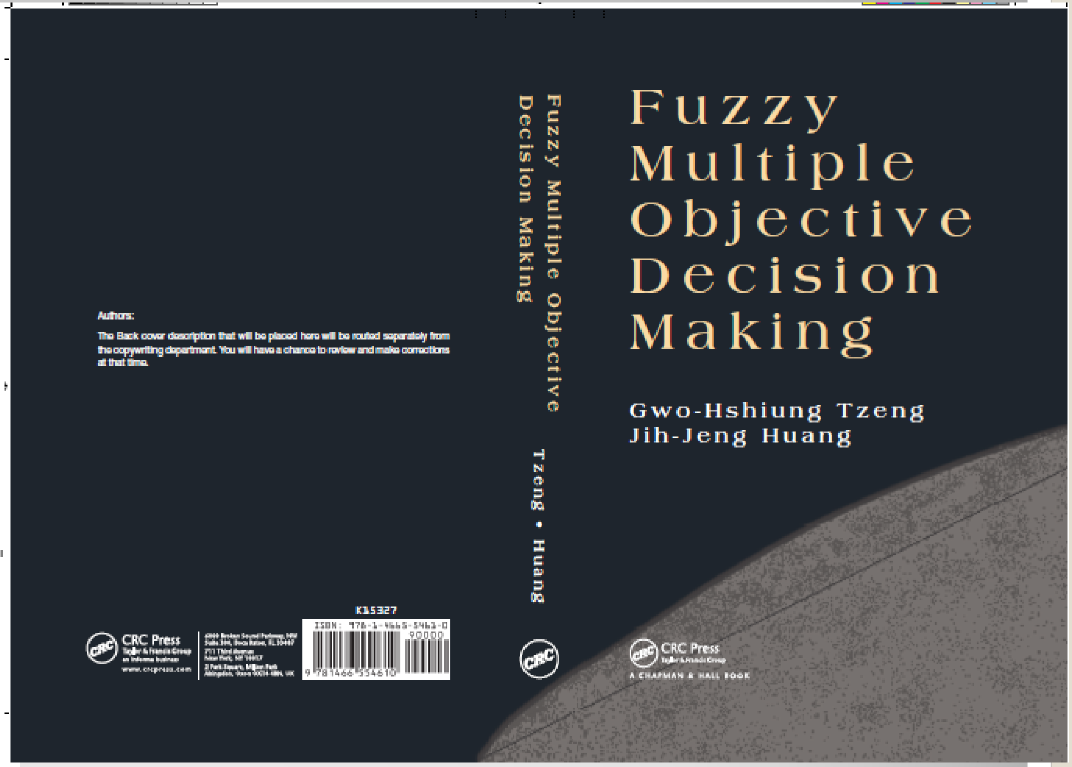
$$\boldsymbol{p}'(\boldsymbol{A}-\hat{\boldsymbol{A}})\boldsymbol{x}+\sum_{i=1}^{n}\sum_{j=1}^{m}p_{ij}^{c}\hat{c}_{ij}+\sum_{i=1}^{n}\sum_{j=1}^{m}p_{ij}^{a}\hat{a}_{ij} \leq B+\hat{B},$$

<extra conditions for $\hat{B}$, p_{ij}^{c}, $\hat{c}_{ij}$, p_{ij}^{a} and $\hat{a}_{ij}$>

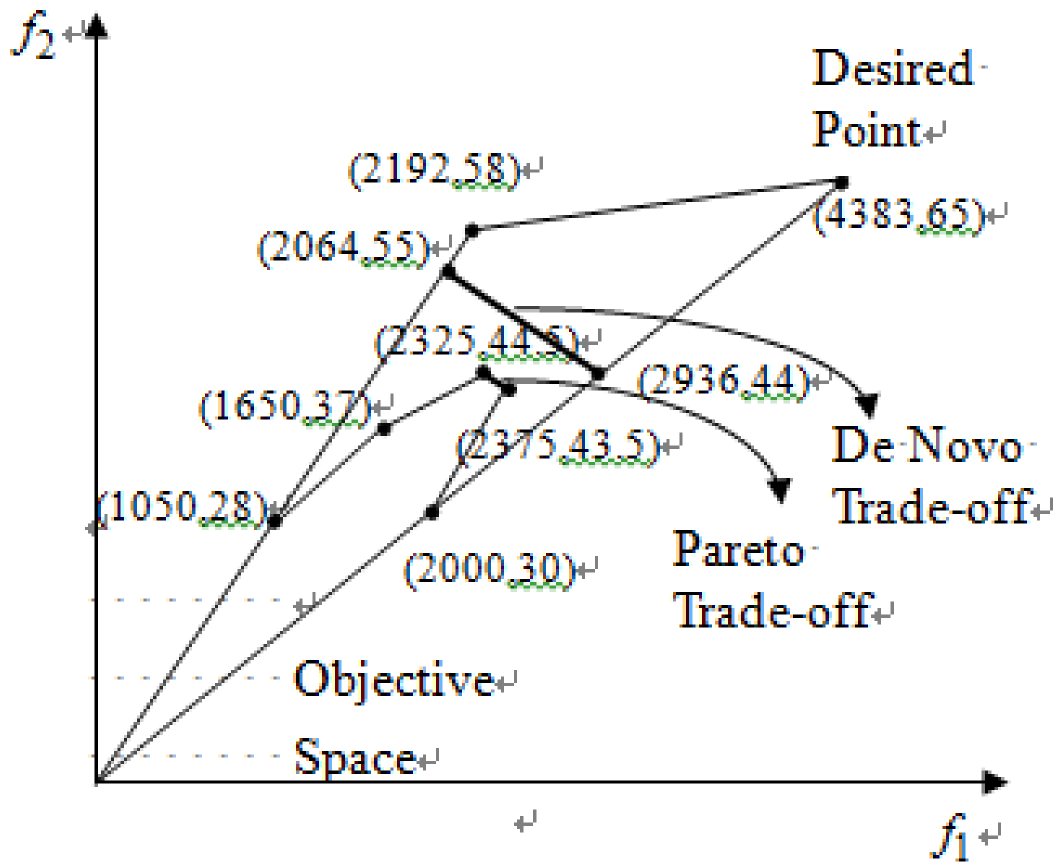
$$\boldsymbol{x} \geq \boldsymbol{0}.$$

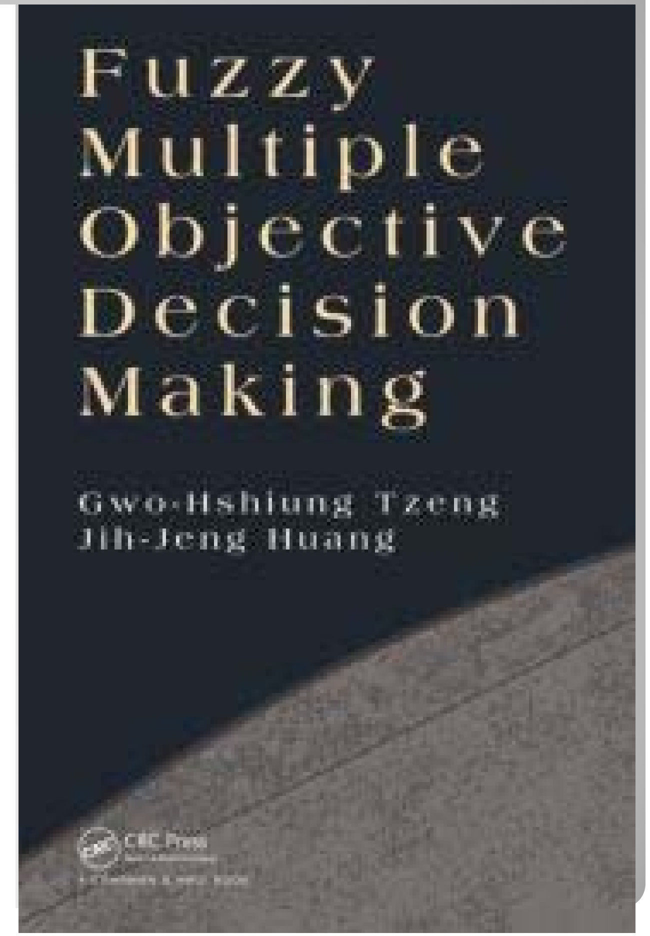
Changeable spaces for achieving the desired point.





A comparison of objective space





Talk

- New concepts and trends of hybrid MCDM model for Tomorrow
- How consider for solving the real world
- Basic concepts of ideas and thinking in trends
- Some examples for the real cases: New hybrid MCDM model
  - MADM: DEMATEL, DANP (DEMATEL-based ANP), Integration (Additive: SAW, VIKOR, Grey Relation Analysis, PROMETHEE, ELECTRE; Non-additive: Fuzzy Integral)
  - MODM: Changeable Spaces Programming
- Conclusions

Conclusions

Conclusions

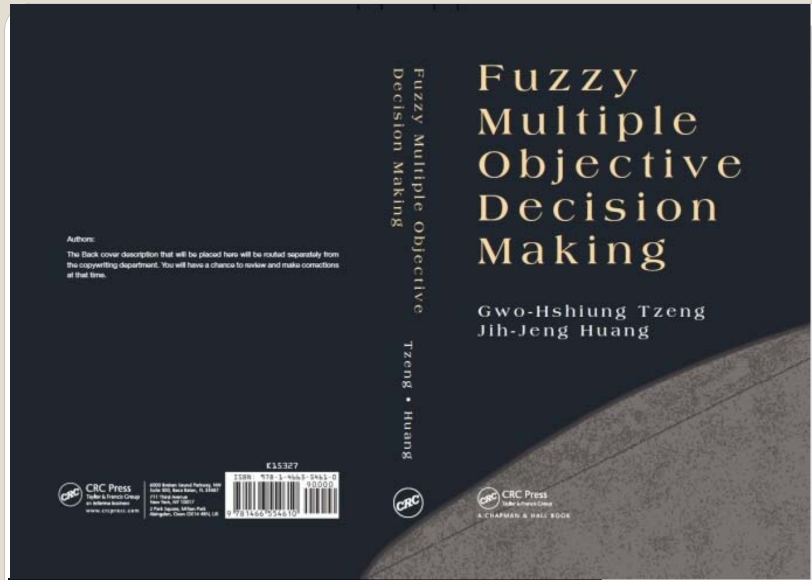
- This study proposed a series of **new Hybrid Dynamic Multiple Criteria Decision Making (HDMCDM) method** in order to overcome the defects of conventional MCDM methods.

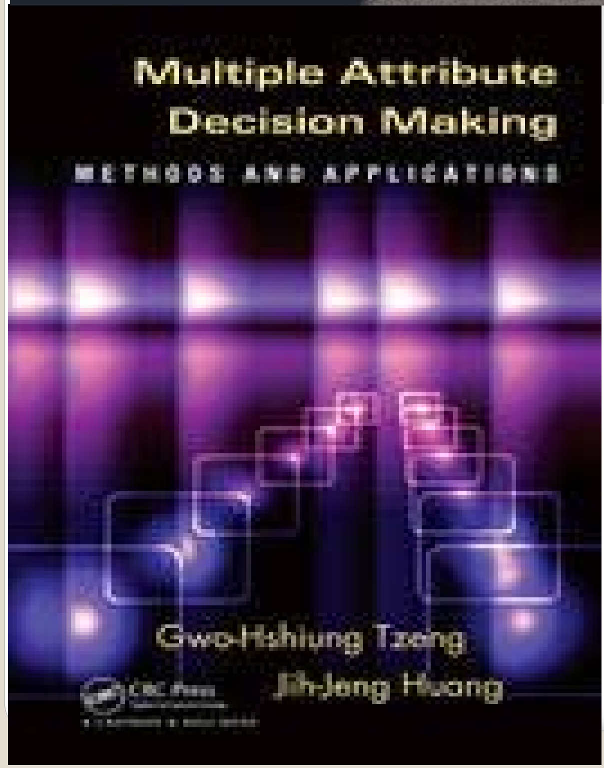
- First, applies the characteristics of influential network relation map (INRM) and influential weights (DEMATEL-based ANP, called DANP) and by using DEMATEL technique to solve interdependence and feedback problems of multi-criteria.

- Second, this study set the best f_j^* values to be the **aspiration level** and the worst f_j^- values as the tolerable level for all criterion functions $(j = 1, 2, \ldots, n)$ to avoid "Choose the best among inferior choices/options/ alternatives".

Conclusions

- Third, this study shifted the concept from the "ranking" or "selection" of the most preferable alternatives to the "**improvement**" of their performances to achieve the aspiration level for each dimension and criterion.
- Fourth, information fusion/aggregation such as fuzzy integrals, basically, **a non-additive/super-additive model**, has been developed to aggregate the performances.
- Finally, we should change basic concepts and thinking from traditional mathematic programming (Goal Programming, Multiple Objective Programming, etc.) into **Changeable Spaces Programming** in future trends





The End

Thank you attention

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